A Post-Keynesian stock-flow consistent model of the Global Financial Crisis and the Age of Secular Stagnation

by

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Statement of originality

I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision.

The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made. I give consent to the final version of my thesis being made available worldwide when deposited in the University's Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.

March 2020

Adam Kaczynski

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Abstract

This thesis is an attempt to build a dynamic, long run, Stock-Flow Consistent, Post Keynesian model of the Global Financial Crisis and Secular Stagnation. While multiple New Keynesian Dynamic Stochastic General Equilibrium models of these historic phenomena already exist, these models are built on theoretical foundations which have been rejected by Post Keynesians because of their inadequacy. The Sraffian Supermultiplier has been chosen as the theoretical framework, isolating parts of the economy generating instability from the parts which may set the trend in the long run. The model uses a continuous-time framework and is expressed as a differential-algebraic system of equations. It is simulated using an Open Source package OpenModelica which is widely used in empirical and technical sciences for simulating dynamic systems. While not calibrated by regression, and more theoretical than econometric, it nevertheless reproduces multiple macroeconomic phenomena and stylised facts which have puzzled mainstream economists. This research is an attempt to advance the macroeconomic modelling methodology and contribute to understanding macroeconomic processes by demonstrating how complex phenomena can emerge when simple parts of the economy interact. The understanding is based on sound macroeconomic theories built by Marx, Keynes, Kalecki, Sraffa and contemporary Post Keynesian economists.

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Table 1: Abbreviations and acronyms

Abbreviation	Definition
CES	Constant Elasticity of Substitution (production function)
DAE	Differential-Algebraic Equations
DSGE	Dynamic Stochastic General Equilibrium
FRED	Federal Reserve Bank of St. Louis Economic Data
GDP	Gross Domestic Product
GFC	Global Financial Crisis
LFT	Loanable Funds Theory
NRI	Natural Rate of Interest
ODE	Ordinary Differential Equations

QTM	Quantity Theory of Money
SFC	Stock Flow Consistent (models)
SSM	Sraffian Supermultiplier

Chapter 1 Introduction

1.1 Aims and Background

The aim of this research project is to contribute to the understanding of the semi-persistent under-utilisation of labour in the US, which has contributed to the growth of poverty and is associated with the recent slowdown of GDP growth. The second aim is to explain the economic mechanisms that lead to periodic recessions, especially in the context of the Great Recession of 2008, also called the Global Financial Crisis (GFC).

Dynamic macroeconomic models should be able to explain the following stylised facts which are depicted in the graphs presented in Section 1.3:

- After the GFC shock, the growth trajectory of real gross domestic product per capita has not recovered to the long-term pre-GFC trend; hysteresis has emerged (Figure 1).
- The rate of change of the stock of household debt is negatively correlated with the rate of unemployment (except for the case of the 2001 recession) (Figure 2).

The following stylised facts help to provide a context for the GFC and the slowdown of economic growth in the US and should assist in model construction:

- The share of labour in the national income has fallen since the 1970s, with growth in real wages and benefits falling below the growth of labour productivity. The gaps between different percentiles in the distribution of household income have widened, as shown in Figure 12 (Blecker, 2016).
- The worsening inequality between the top and bottom income distribution groups has contributed to the weakening of the link between disposable income and household spending (Cynamon & Fazzari, 2015).
- This weakening was mitigated in the late 1990s and early 2000s by spending financed by household borrowing, as shown in Figure 2 (Blecker, 2016).

- The housing bubble financed by excessive borrowing was caused by the unusually low interest rates in the period 2002-2006, as shown in Figure 8 (Taylor, 2009; Cynamon & Fazzari, 2015). The process of securitisation which contributed to lowering the mortgage rates is discussed in Section 2.5. There was wide bipartisan support to expanding house ownership among all social groups, what led to lowering lending standards (the sub-prime lending).
- The Financial Crisis was largely caused by the bursting of the housing bubble, leading to loss of equity, delinquencies and foreclosures (Figure 3, Figure 6 and Figure 7). The financial sector was vulnerable because of the deteriorating quality of risk management (Bernanke, 2010).
- The Financial Crisis of 2007 marked the end of significant leveraging of US households (Blecker, 2016).
- In the period after the GFC, policies aimed at reducing the nominal rate of interest to ultra-low values did not restore full employment (Summers, 2014).
- During the current recovery the civilian unemployment rate fell predominantly because of a falling labour participation ratio, but employment has only partially recovered (Figure 13).

The explanations of what went wrong within the financial sector in the lead-up to the crisis of 2007 provided by Ben Bernanke (2010), John Taylor (2009), Lawrence Summers (2014) and other leading New Keynesian economists have not been questioned by heterodox economists. This includes the impact of inadequate risk management, lowering of lending standards (subprime lending), securitisation, and the growth of shadow banking system. This is discussed in Section 2.5.

What might be questioned is whether the New Keynesian theoretical framework has afforded a satisfactory explanation of the transmission mechanism responsible for spreading disruption from the financial to the real sphere of the economy and whether there were other, deeper and less obvious causes of the Great Recession, which began to destabilise the real economy before the financial system became affected. A more detailed discussion of these issues is provided in Sections 2.4 and 2.6.

In this thesis the recession of 2008 is called the Global Financial Crisis (GFC), as this name is commonly used, but the focus is on the changes in the real economy rather than on the processes within the financial sector itself.

More research is needed into the macroeconomics of the GFC and more work is needed to improve macroeconomic modelling methodologies in general. A significant number of macroeconomic models exist within the Post Keynesian (heterodox) tradition which might offer alternative insights into the mechanics of economic stagnation and the business cycle, but a gap still exists in demonstrating how the financial crisis morphed into a deep global recession. This thesis is attempting to clearly demonstrate the causal mechanism linking income distribution, fiscal policy and debt-financed household expenditure with two main macroeconomic phenomena of recent decades – the GFC and Secular Stagnation.

1.2 Research Objectives and Methodology

This thesis will attempt to address the following research questions:

- How was the economic growth trajectory (Figure 1) affected by the changes in income distribution between different income groups (Figure 12) (Bhaduri & Marglin, 1990; Onaran & Galanis, 2014; Stiglitz, 2015; Cynamon & Fazzari, 2015; Blecker, 2016)?
- How did the real estate bubble (Figure 2, Figure 3, Figure 7) affect the productive economy in the medium and long run and how did it differ from a stock market bubble (Figure 4)?
- How did the financial crisis morph into a deep global recession, what was the principal transmission channel?
- Is it possible to reconcile the theory of debt-deflation with theories of effective demand (Fiebiger, 2017; Fiebiger & Lavoie, 2017)?
- How effective can monetary (Figure 8) and fiscal (Figure 10) policy be in the context of a deep recession caused by the collapse in aggregate demand (Godley & Lavoie, 2007)?

These questions will be addressed by creating a dynamic macroeconomic model allowing for simulating of scenarios which illustrate the transition of national economies in highly-developed countries (particularly the US) from the high-growth, low-unemployment macroeconomic environment of the early post-war era to the recent low-growth and less stable contemporary environment. The model should simulate the impact of debt-financed real estate investment during the housing bubble and subsequent debt deleveraging during the GFC by applying a Stock-Flow Consistent methodology which combines short-run and long-run analysis.

The macroeconomic framework chosen to model the GFC is the Sraffian Supermultiplier, as presented by Serrano and Freitas (2017) and further developed by Fiebiger and Lavoie (2017). The Supermultiplier works in the short- and medium-run by combining the effects of the Keynesian spending multiplier, driven by changes in autonomous expenditure, and the investment accelerator (Nikiforos, 2018). The rate of accumulation (depending on corporate investment) adjusts to maintain the desired capital utilisation rate.

The long-run growth trajectory simulated in the model will mostly depend on endogenous processes within the private sector, such as the accumulation of firms capital, household wealth and growth in productivity. These are affected, in turn, by the distribution of gross income between wages and profits and on the relative size of government expenditure.

Since modelling was undertaken prior to March 2020, the economic consequences of the COVID-19 pandemic of 2020 (ongoing in December 2020) are not included in the model.

1.3 Graphs illustrating recent macroeconomic phenomena and dynamic processes observed in the American economy

The following graphs provide the context to the research questions and illustrate "stylised facts" mentioned above in Section 1.1. Some variables from the graphs will be used as "exogenous variables" in the models described in Chapter 4. The dynamic models will simulate at least some of the trajectories depicted in the graphs such as GDP growth or changes in the unemployment rate.

1.3.1 Real GDP growth trajectory

Two main recent phenomena seen in the real GDP graph are the deep recession of 2008 and the stunted growth after the crisis (identified as a “secular stagnation”). Previous recessions had been followed by a temporary increase in the real GDP growth rate so the economy would have recovered the GDP losses. When plotted in a semi-logarithmic scale, the GDP trajectory after 2010 appears to run almost parallel to the line of the trend growth before 2008.

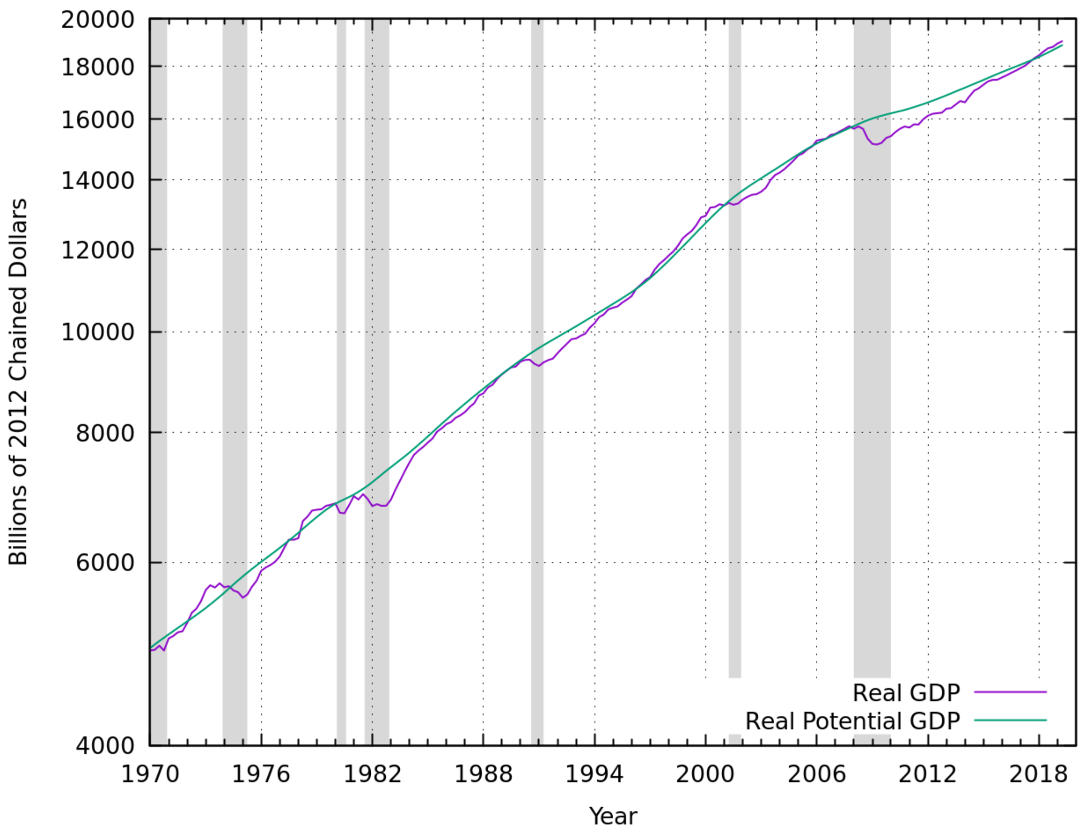

Figure 1: GFC and the "secular stagnation".

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (GDPC1), U.S. Bureau of Economic Analysis (GDPPOT) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.2 Net lending to households and unemployment rate

Except for the recession of 2001 all the recessions (and resulting increases in unemployment rate) were preceded by large increases in the rate of net lending to households and overlap with the drops. The drop in the rate of net lending after 2006 preceded the increase in the unemployment rate during the GFC, which may be explained by causal dependence.

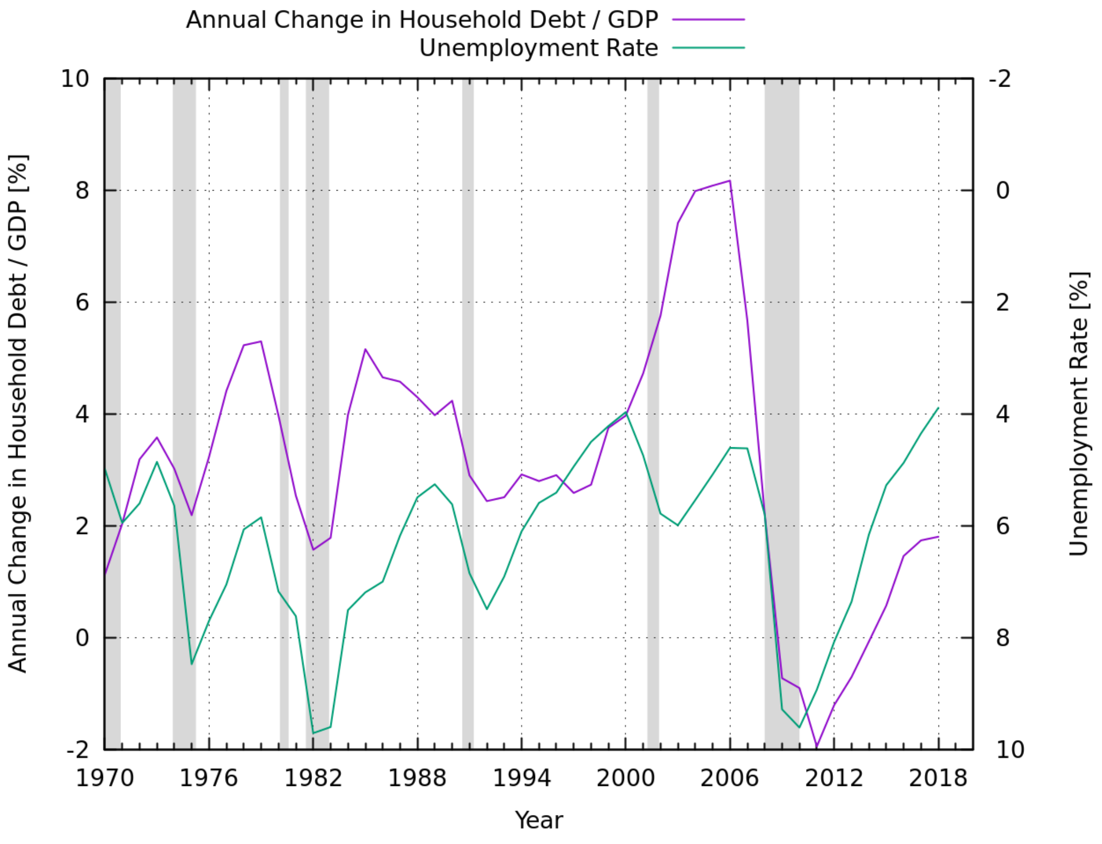

Figure 2: Net lending to households and unemployment rate.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US) (CONSUMER, HHMSDODNS), U.S. Bureau of Economic Analysis (GDP), U.S. Bureau of Labor Statistics (UNRATE) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.3 Total private fixed investment and its components

All recessions since 1974 overlap with significant drops in total private investment.

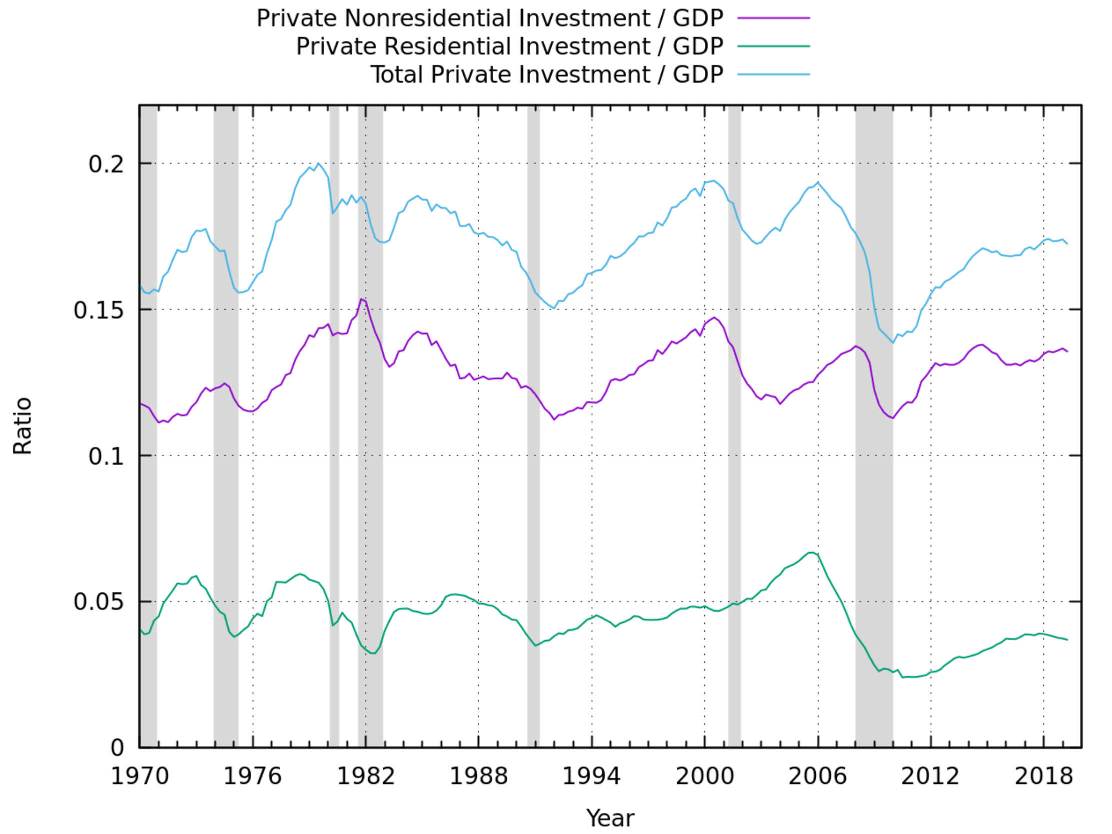

Figure 3: Private fixed investment.

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (PNFI, PRFI, GDP) and The National Bureau of Economic Research (USREC).Shaded areas indicate U.S. recessions.

1.3.4 The evolution of Tobin's q ratio

The highest peak on the q ratio graph corresponds to the dotcom bubble. The recession of 2001 followed the crash. The growth of the stock market bubble was linked with an increase in corporate (fixed capital) investment. When the stock market bubble burst, corporate nonresidential investment fell significantly but (as seen on the graph above), private residential investment eventually compensated for the losses (until the peak of 2006).

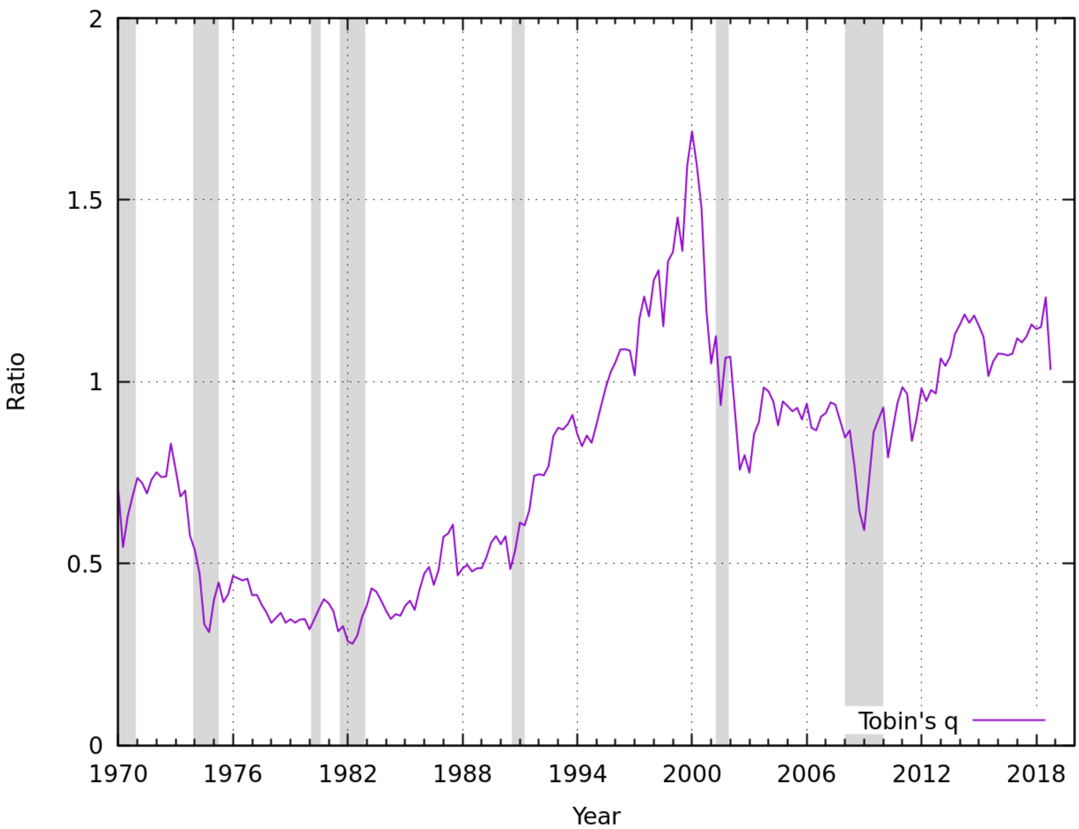

Figure 4: The evolution of Tobin's q ratio.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US) (NCBEILQ027S, TNWMVBSNNCB) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.5 The evolution of real productivity

Productivity growth has slowed down after the GFC. This appears to be one of the causes of the drop in the rate of GDP growth.

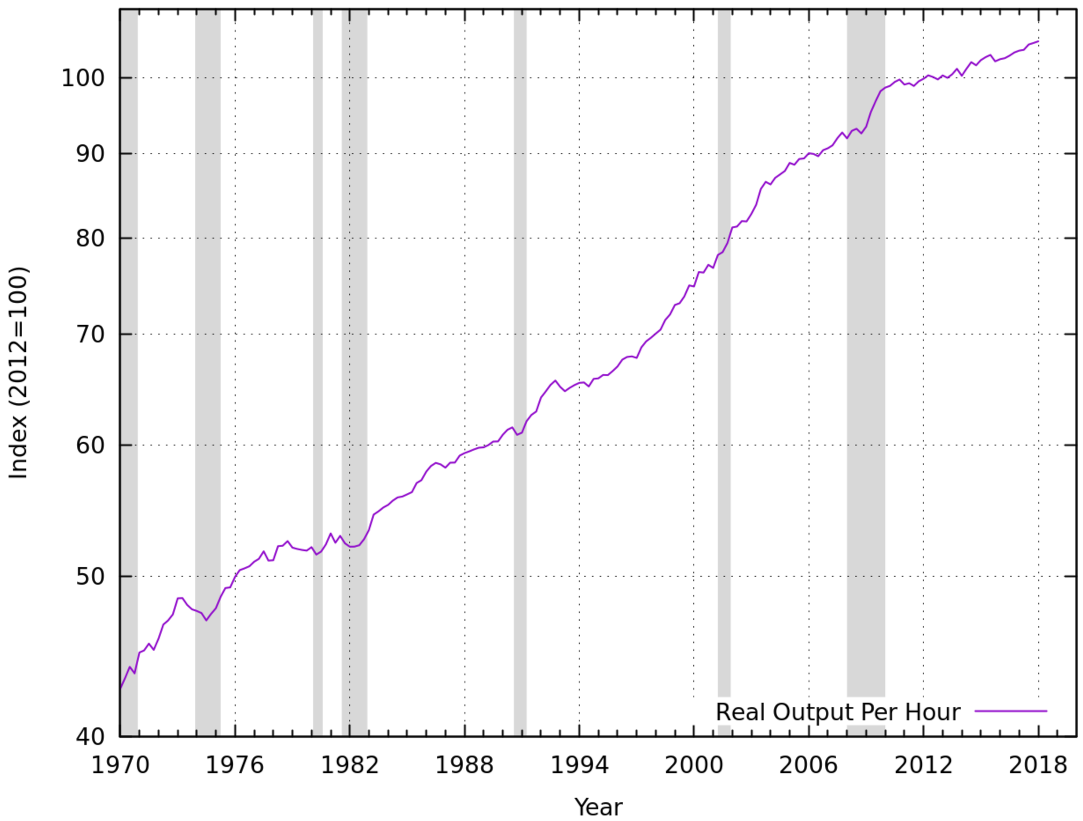

Figure 5: Productivity

Real Output Per Hour of All Persons (Nonfarm Business Sector).

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Labor Statistics (OPHNFB) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.6 The ratio of the value of real estate to mortgages

The ratio of real estate value (including the land and actual buildings) to the stock of mortgages was relatively stable since the early 1990s, excluding the dip which occurred when the real estate bubble started bursting in 2006.

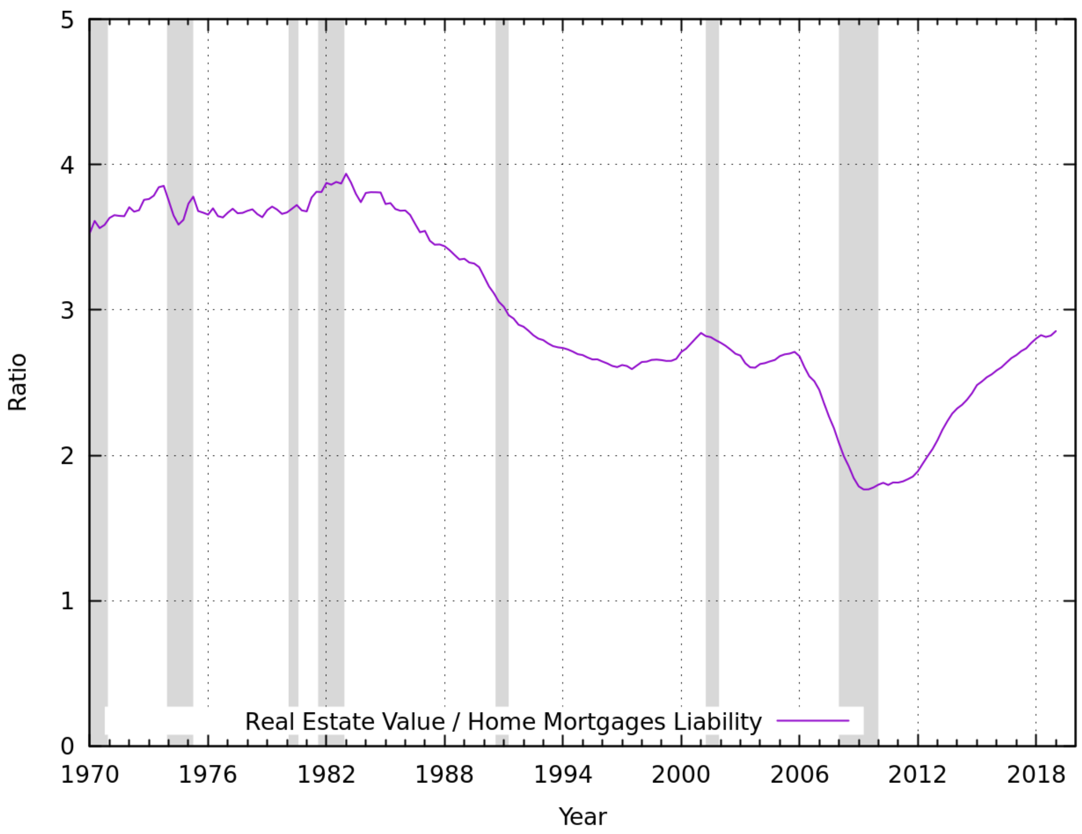

Figure 6: Ratio of value of real estate to level of home mortgages liability.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US) (HNOREMQ027S, HMLBSHNO) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.7 Assets and liabilities of households

The changes in the ratio of real-estate value to GDP during the housing bubble and the GFC were the main drivers of the changes in total household wealth to GDP ratio.

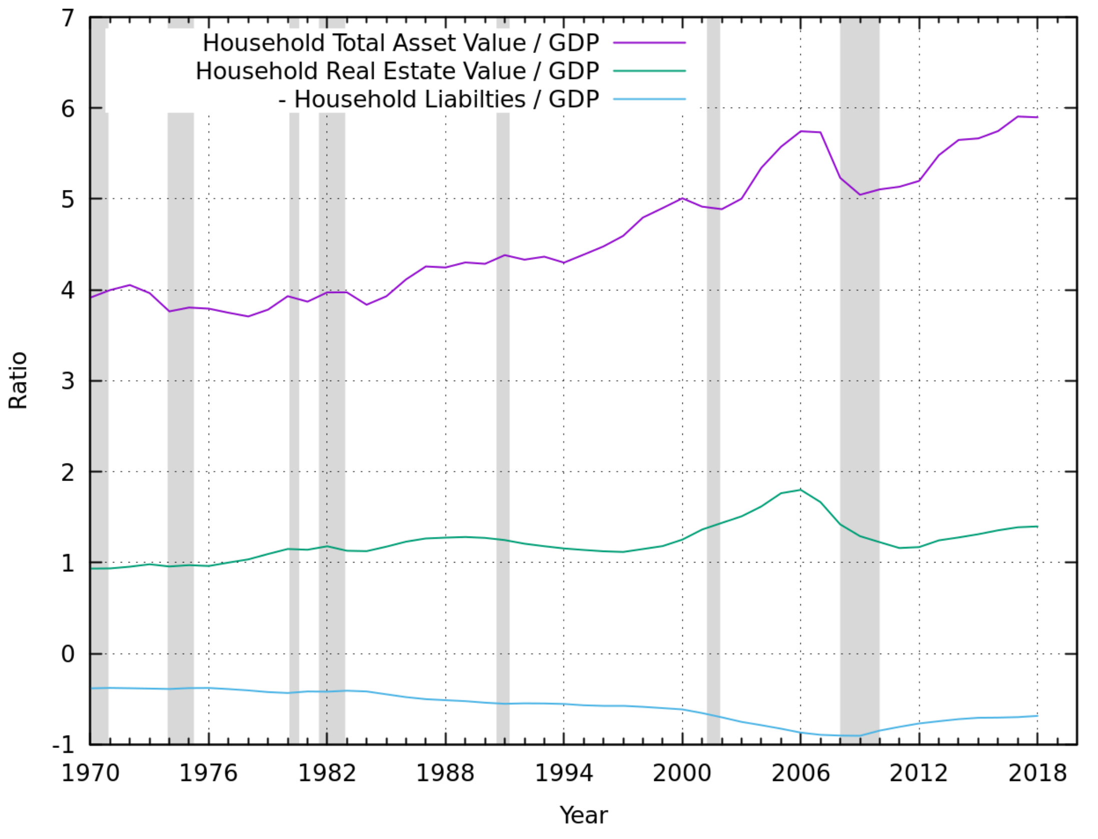

Figure 7: Assets and liabilities of households.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US) (TABSHNO, HNOREMV, HMLBSHNO, CCLBSHNO), U.S. Bureau of Economic Analysis (GDPA) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.8 The inflation rate and the interest rate

In response to the financial crisis which developed in the second half of 2008, the Federal Reserve lowered interest rates to near-zero and started expanding the balance sheet of the central bank, providing liquidity to commercial financial institutions. They also engaged in setting the prices of various financial assets such as longer maturity Treasury bonds and mortgage based securities through quantitative easing. This set of implied rates of return on assets prevented further capital losses. The impact on yield curve was supposed to increase investment and lending.

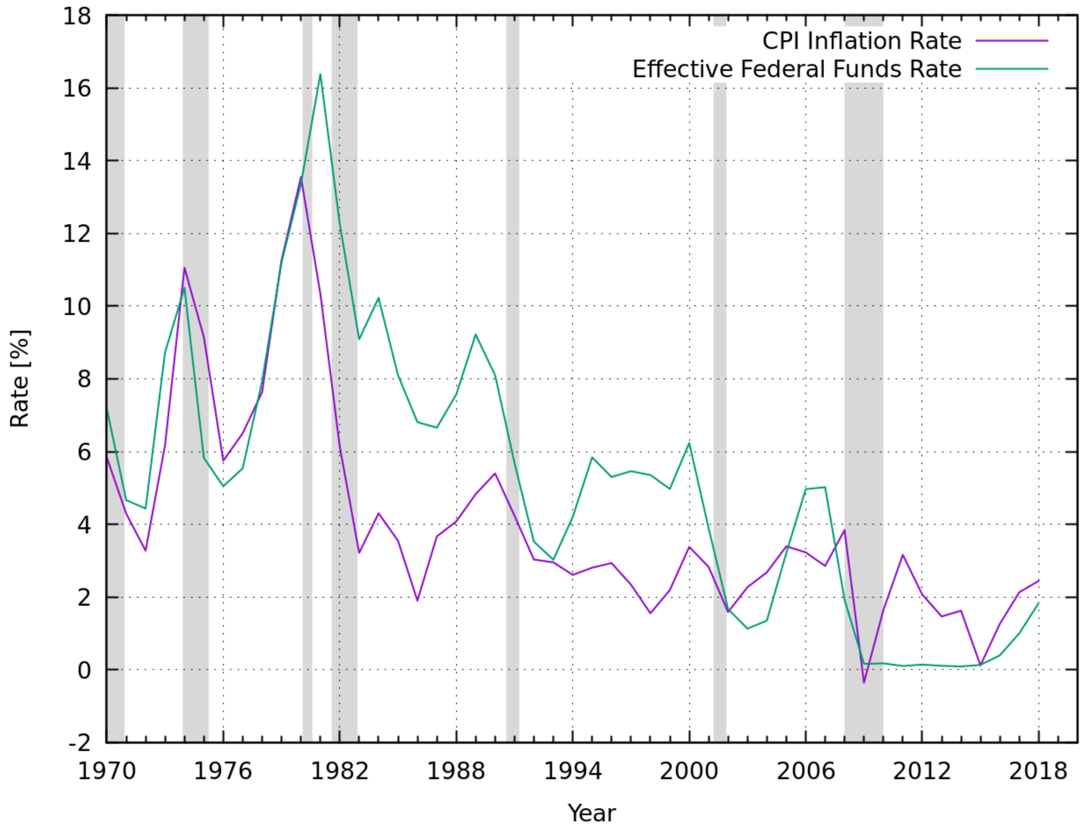

Figure 8: Inflation and effective federal funds rate.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US) (FEDFUNDS), World Bank (FPCPITOTLZGUSA) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.9 The sectoral balances

Private sector net lending displays counter-cyclical behaviour while public sector net lending is pro-cyclical (public sector deficits shrink when there is high level of economic activity of the private sector). The fall in the private sector net lending in the period 1995-2006 and a dramatic reversal of this trend in 2008 could be linked with the credit expansion and subsequent debt deleveraging (illustrated in Figure 2). The current account balance also displays weak counter-cyclical behaviour similar to private sector net lending. All the balances have to sum up to zero.

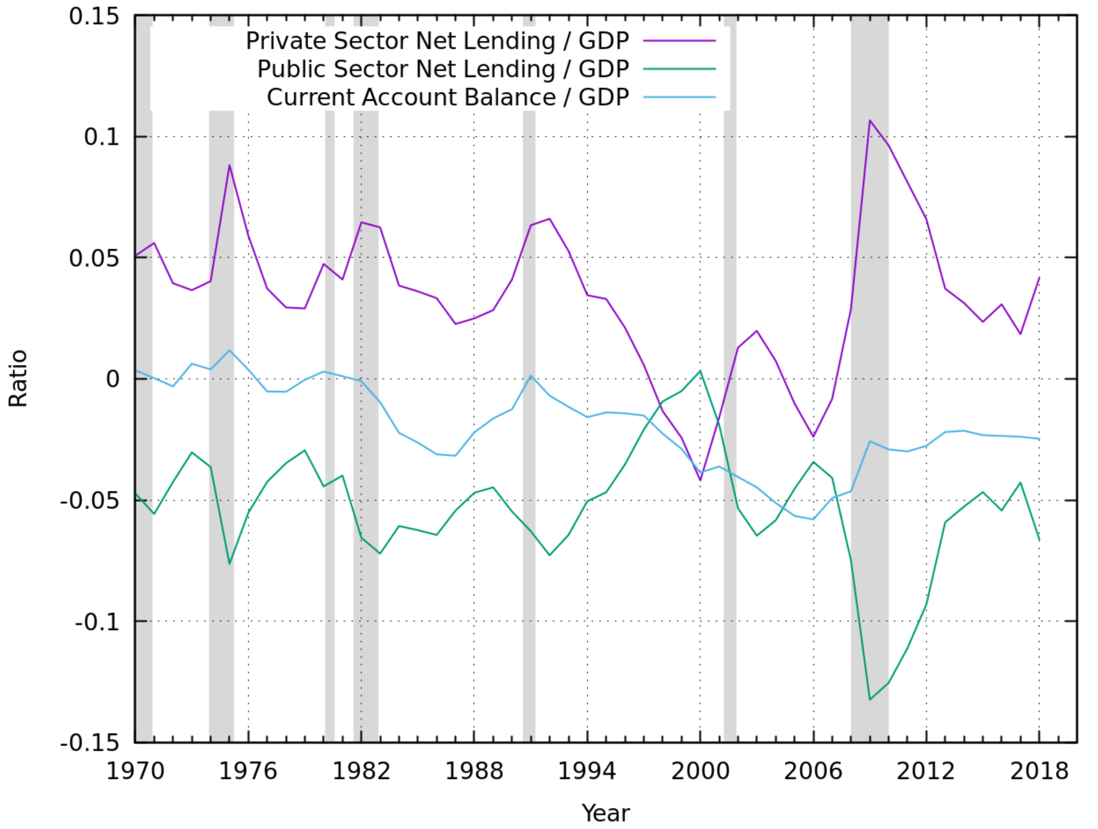

Figure 9: Sectoral balances (using NIPA methodology).

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (W994RC1Q027SBEA, AD01RC1Q027SBEA, NETFI, GDPA) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.10 Changes in the government expenditures

The decrease in the government expenditures after 1991 can be mostly explained by the fall in defence spending. During the GFC expenditures were increased as a part of the stimulus program. There is a long-term downward tendency in the spending to GDP ratio.

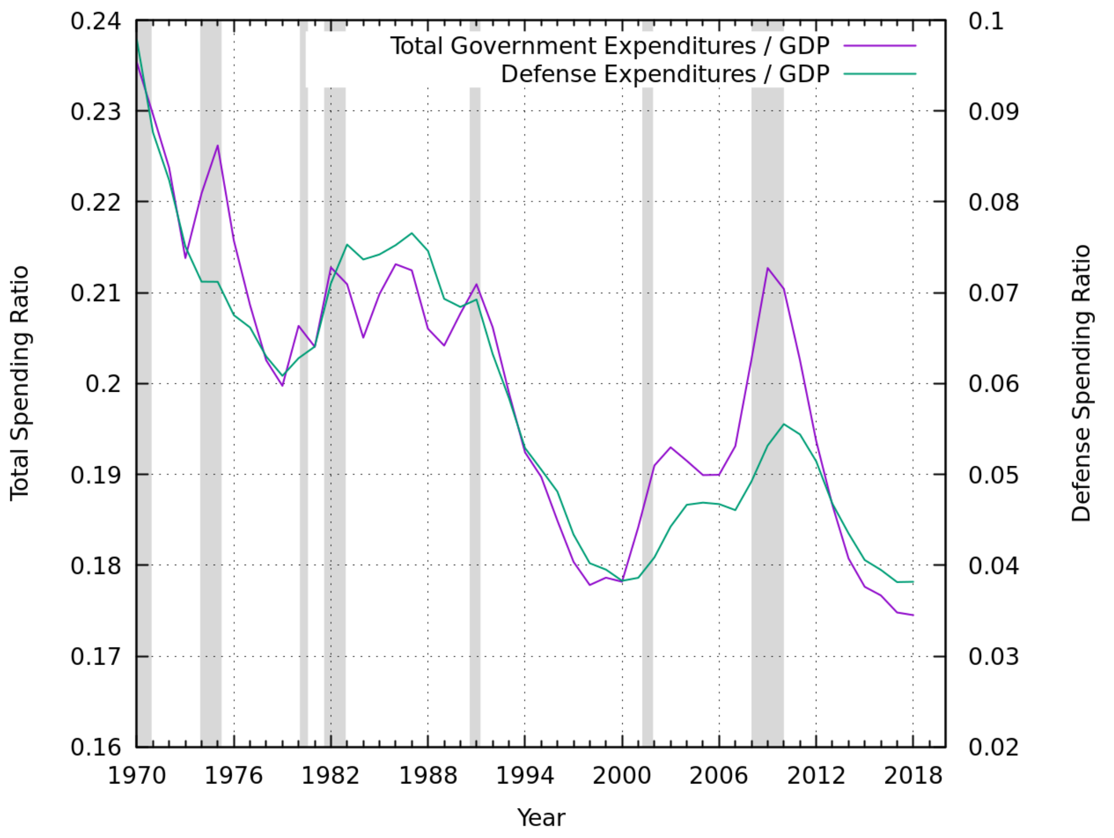

Figure 10: Government consumption and gross investment expenditures to GDP ratio.

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (GCEA, FDEFX, GDPA, GDP), Chinn (2013) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.11 Changes in the personal saving rate

The personal saving rate was the lowest during the period of increased mortgage borrowing (2000-2006). The saving rate increased again during the GFC. These changes seem to be superimposed over a slowly changing trend, possibly related to behavioural changes.

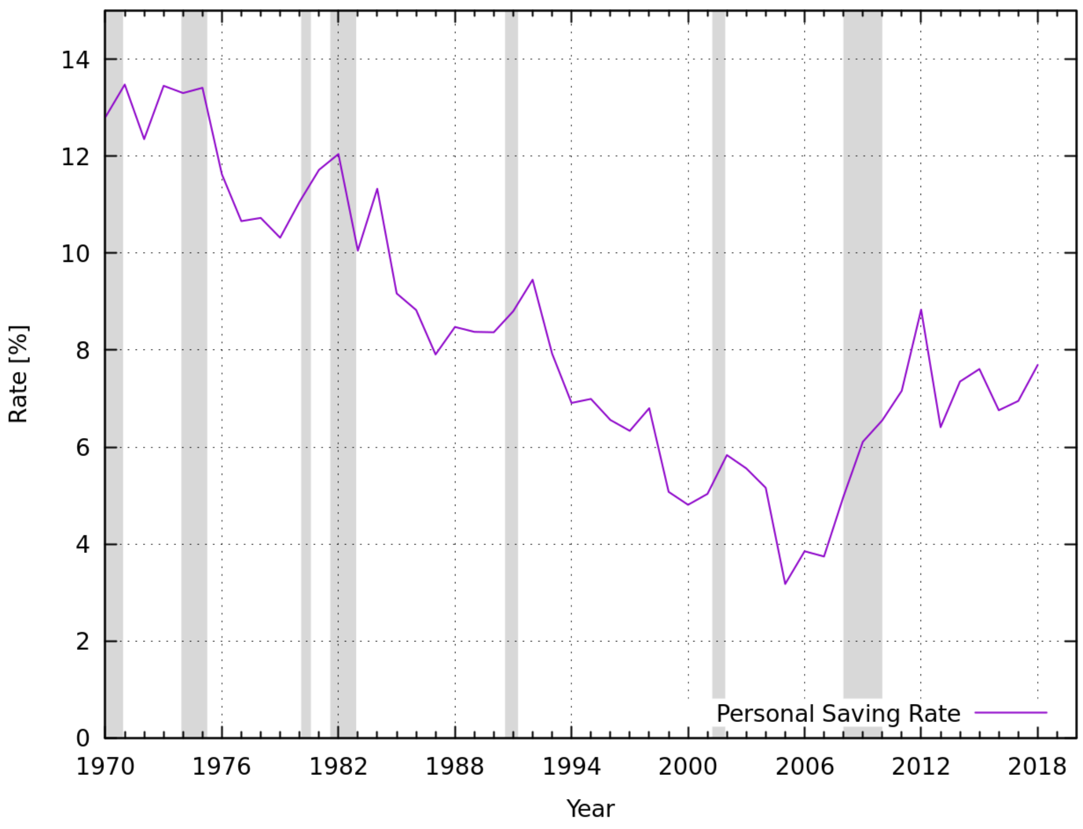

Figure 11: Personal saving rate.

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (PSAVERT) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.12 Changes in the share of post-tax income of top income groups

Changes in disposable income distribution were one of the most significant processes affecting American society in the last 40 years. It can be argued that the richest (the top "1%") gained the most. Changes in wealth distribution followed the changes in income.

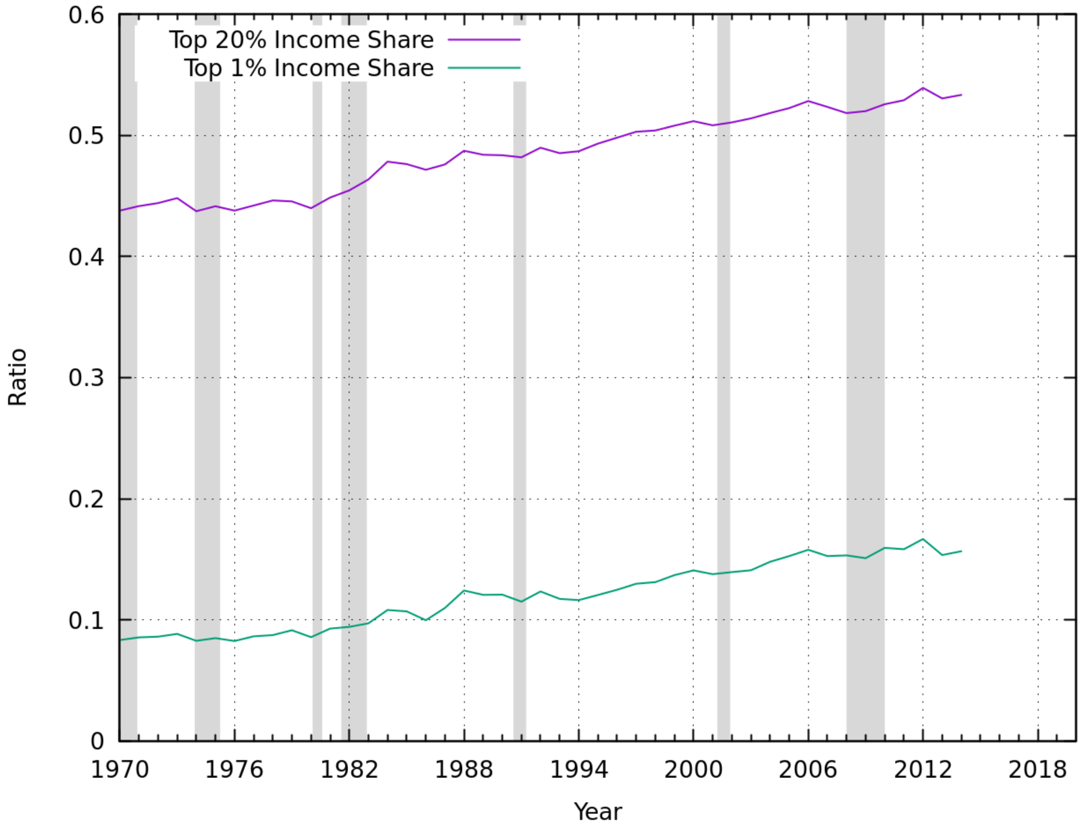

Figure 12: Share of top income groups in post-tax national income.

Based on the data from World Inequality Database (USA, sdiinc992j, p80p100 and p99p100) and Federal Reserve Economic Data, The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.13 Population and employment

While the size of the working age population has been affected by demographic processes such as ageing, the growth rate of the civilian labour force fell as a result of the rise in the number of discouraged workers during and after the GFC.

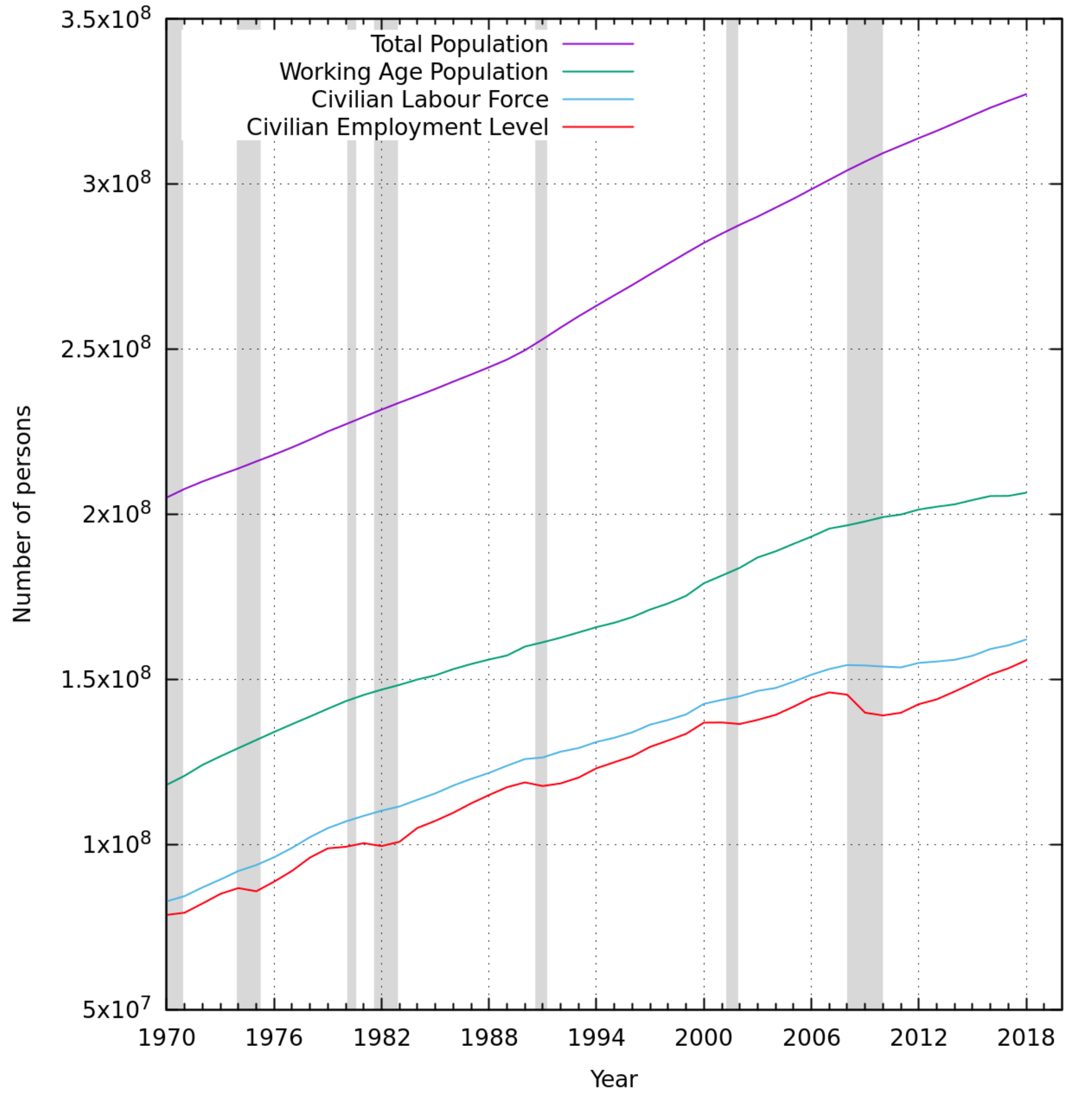

Figure 13: Population and employment.

Based on the data from Federal Reserve Economic Data, World Bank (POPTOTUSA647NWDB), OECD Main Economic Indicators (database), http://dx.doi.org/10.1787/data-00052-en (LFWA64TT), U.S. Bureau of Labor Statistics (CLF16OV, CE16OV) and The National Bureau of Economic Research (USREC). Shaded areas indicate U.S. recessions.

1.3.14 Summary of the processes identified in the graphical analysis

The graphs provided in this section illustrate several processes which have occurred in the American economy over the last 50 years and support stylised facts listed in Section 1.1. The main drivers of the changes in the macroeconomic parameters due to their impact on the aggregate demand, have been:

- rising income inequality - Figure 12
- the reduction of government expenditure to GDP ratio (partially caused by the reduction of defence expenditures after the end of the Cold War) - Figure 10
- the rise and fall in debt-financed household expenditure - Figure 2 and Figure 3
- changes in the valuation of company equities - Figure 4

Some variables have been changing on their own in the long run but were also affected by short-run changes in the aggregate demand. These include:

- changes in the trade balance - Figure 9
- changes in the real productivity - Figure 5
- changes in the personal saving rate - Figure 11
- changes in the population and employment - Figure 13

The changes in the parameters listed above affected the real GDP growth trajectory shown in Figure 1.

1.4 Thesis overview and original contribution

Chapter 2 of the thesis provides a brief review of literature which is relevant to addressing the research objectives. The fundamental differences between the neoclassical and Keynesian schools of economic thought are outlined and analysed in Sections 2.1 and 2.2. The debate about the methodology used for describing and modelling the financial markets between the supporters on Loanable Funds Theory and the Keynesians is highlighted in Section 2.3. This debate has still not been resolved as New-Keynesians have resurrected the Loanable Funds Theory which is applicable in the long run in a neo-Wicksellian form. The mechanism responsible for creating the market for loanable funds in New Keynesian theoretical framework is the existence of the

Natural Rate of Interest and assumed high sensitivity of investment to the interest rate (Woodford, 2003). A brief critique of the elements of the New Keynesian theory related to Loanable Funds Theory in the functioning of financial markets is included in Section 2.4. A Post Keynesian alternative to the Loanable Funds Theory is also presented.

The second part of Chapter 2 contains a review of the theoretical explanations of the GFC and the Secular Stagnation. While New Keynesians concentrate on highlighting the imperfections in functioning the financial markets and shocks leading to the fall of the Natural Rate of Interest, Post Keynesians highlight the consequences of falling aggregate demand. The Sraffian Supermultiplier model of the business cycle which provides the theoretical framework for the models built in this research project is described in this chapter.

Chapter 3 provides a description of the Stock Flow Consistent methodology chosen for building models in the thesis. This methodology is contrasted with the methodology used for building Dynamic Stochastic General Equilibrium (DSGE) models. The reasons for choosing the continuous-time framework instead of the more common discrete-time are presented. The difference between systems described by Ordinary Differential Equations and Algebraic-Differential Equations is highlighted.

The Stock Flow Consistent model of the Global Financial Crisis and Secular Stagnation is defined in Chapter 4. The calibration procedure and some calibration data is included. The results of simulation are presented on multiple graphs. This is followed by the analysis of the results. The similarity of the simulated trajectories and the reference data illustrated on the graphs presented in Section 1.3 is discussed. Finally the limitations of the model and the possible future directions of further development of the models are presented.

Chapter 5 contains the conclusions of the thesis and suggests possible future work based on the research presented in this document.

The following are the elements of novelty and originality in the research conducted for this thesis:

- This is the first attempt to build an integrated dynamic SFC model simulating both the long-term impact of rising income inequality on growth and the medium-term impact of debt-financed household spending and debt-deflation, driving the business cycle. The study highlights the causal link between the inequality and the instability of the financial system caused by excessive debt accumulation (Stiglitz, 2015).
- This study demonstrates in a convincing way that the causal mechanism of the GFC was the fall in aggregate demand when households stopped borrowing and started deleveraging. Evidence suggests that this demand shock was then amplified by the supermultiplier effect.
- The study also demonstrates the emergence of hysteresis as a consequence of slower capital accumulation and a fall in productivity growth during and after the crisis.
- Large demand-driven SFC models have been only built in a discrete-time framework which makes them difficult to analyse and modify. A continuous-time framework is used to address these limitations. This contributes to the development of SFC modelling methodology and provides a link to a more mature modelling methodology used in empirical science and engineering.

Chapter 2 Literature Review

2.1 The Debate between Post Keynesian and Neoclassical Economics

The definition of who can be called a Post-Keynesian was a subject of a debate between Paul Davidson who proposed a narrow definition and John King and Marc Lavoie who proposed a wider one. Davidson (2005, 2006, 2007) identified salient features present in the schools of economic thought which originated from Keynes's General Theory's principle of effective demand, arguing that Keynesian thinking was diluted and eventually rejected by the economists pretending to be "New Keynesians". This happened by attempting to reconcile some of the ideas borrowed from the General Theory with the neoclassical framework. But the neoclassical framework is, according to Davidson, less general than the Keynesian one. The modern neoclassical framework is supposed to follow the strict formalism present in mathematics and is built upon several axioms which had been already present in pre-Keynesian neoclassical economics. According to Davidson, Keynes rejected the "gross substitution axiom" and the "neutrality of money axiom" because he did not consider these axioms to be consistent with the reality. Another area of disagreement between Keynes and early neoclassical economists was related to the concept of "uncertainty".

The meaning of the gross substitution axiom is that "everything is a good substitute of everything else" (Davidson, 2007). With this axiom it can be demonstrated that all the markets (including the labour market) will clear if all the prices are instantaneously flexible. This is the state of "general equilibrium" in the sense defined by Walras and refined by McKenzie, Arrow and Debreu. If this axiom is removed, the existence of general equilibrium cannot be proven even with wage and price flexibility. Keynes explicitly rejected this axiom for the substitution between the demand for money and the demand for other commodities:

> *The second differentia of money is that it has an elasticity of substitution equal, or nearly equal, to zero; which means that as the exchange value of money rises there is no tendency to substitute some other factor for it; — except, perhaps, to some trifling extent, where the money-commodity is also used in manufacture or the arts. This follows from the peculiarity of money that its utility is solely derived from its exchange-value, so that the two rise and fall pari passu, with the result that as the exchange value of money rises there is no motive or tendency, as in the case of rent-factors, to substitute some other factor for it. (Keynes, 1936, p 231)*

Since the gross substitution axiom is not applicable to the real world, the economy may not be in the state of general equilibrium and some agents may persist in hoarding money regardless of how low prices of products fall.

The money neutrality axiom postulates that changes in the quantity of money only affect nominal variables (these which are measured in monetary units), without affecting real variables such as real GDP, employment, rate of growth, etc. While short-term violations of money neutrality is acknowledged by many economic schools, long-term neutrality of money remains one of the basic assumptions made by neoclassical economics. Keynes (1933) rejected the axiom of money neutrality in a short article "A Monetary Theory of Production" written before he completed "The General Theory". According to Keynes, booms and depression (real phenomena, affecting the long-term growth path of the economy) occur in an environment where money is not neutral. Keynes distinguished between a "real-exchange economy" where money only appears as a neutral link between real transactions (this is the economy analysed and modelled by neoclassicals where only some insignificant deviations are allowed) and a "monetary economy" existing in the real world. Contracts between agents are always expressed as nominal (monetary) values. Keynes pointed out the fact that changes in value of money affect the real value of debts and that the rate of interest affects the behaviour of agents.

Intertemporal utility maximisation is only possible if the future can be predicted. Keynes rejected the idea that agents, no matter how smart or "rational" they are, can predict the future. Davidson (2007) highlighted the inability of entrepreneurs to statistically assess potential future gain based on existing data sets. According to Keynes (1936), it is the uncertainty about the future that leads agents to store some of their non-consumed income (savings) in the form of liquid assets. The value of saving planned in the current time period in the form of liquid financial assets does not need to coincide with the value of debt-financed investment financed planned by entrepreneurs. If some of the income is not spent on products, the stock of unsold products will rise, sending a signal to firms to scale down production in the next period. Uncertainty about the future means that entrepreneurs cannot apply probability calculus to help determine the best investment strategy. According to Keynes, all that remains as a driving force of investment are "animal spirits" or what is now called the level of business confidence (conveniently estimated by some financial institutions as an "index"). When the level of business confidence is low, the preference for liquid assets increases, also as a precautionary measure. As a consequence, spending on newly produced capital assets decreases which further depresses aggregate demand and reduces the level of business confidence even further. When the economy recovers, the same process occurs in the opposite way, leading to an acceleration in investment. The economy flips between two states – a state of diminished growth and pessimism and a state of booming growth and euphoria.

Keynes rejected Say's law presented as *"the aggregate demand price of output as a whole is equal to its aggregate supply price for all volumes of output"* (Keynes, 1936 p. 26). The consequence of Say's law is that if someone makes an extra unit of a product when the supply increases, the aggregate demand increases by the same value and the product is sold. All the income from the current period (wages, profits, rents) is spent on some products during the current period or in the near future. The economy remains in a state of global equilibrium (prices and wages may obviously change so that all the markets clear but there are no unsold products made in the current period). This implies that all the workers supplying labour are employed. In a world described by Say's law agents who abstain from spending (save) provide funds for the agents who wish to borrow on the loanable funds market in order to finance their investment. The volume of investment is determined by the volume of saving. The level of economic activity is limited by the supply of labour and the economy operates at a near-full capacity utilisation level. Davidson (2006, p.147) states that *"agents who plan to buy the product of industry in the current period are not required to earn income currently or previously to their exercise of spending in today's market"*. Davidson (2007) also mentions that for Keynes Say's law is a special case covered by his General Theory if the neoclassical axioms are added. But, according to Keynes, we do not live in a world where Say's law would be applicable in principle. On the other hand, neoclassical and New Keynesian economists would attribute limited and temporary violations of Say's law to imperfections in the functioning of certain markets and exogenous shocks. Say's law holds in the long run. Bernanke (2000) considered the malfunctioning of the loanable funds market (a deflation-induced financial crisis) and the malfunctioning of the labour market (real wages increasing above market-clearing levels) as possible mechanisms transmitting nominal shocks to the real economy, leading to events such as the Great Depression or the Global Financial Crisis.

2.2 Differences in the understanding of "economic equilibrium" between neoclassical and Post Keynesian economists

The concept of economic equilibrium requires further clarification. What is considered to be a general (Walrasian) equilibrium is defined as a stable state in which supply matches demand on all the markets (Lavoie, 2014). In the absence of external disruptions the economy will forever remain in the state of equilibrium. Therefore this state is a long run, steady state or stationary equilibrium (Godley & Lavoie, 2007). Static models such as the IS/LM are defined in terms of a static equilibrium which depends on exogenous parameters. The neoclassical growth models are built upon microfoundations containing rational expectations and intertemporal optimisation (maximisation of the sum or integral of the representative agent's discounted utility function). Since the future growth path is known in the absence of exogenous shocks, the long-run equilibrium can be determined and has the same meaning as in the static models. According to the New Keynesian school, the actual economy is subjected to stochastic productivity shocks and some of the market does not clear instantaneously or is distorted. The system may show dynamic (transitional) behaviour on its path towards the long run stable state. This is the meaning of the term "dynamic equilibrium" in DSGE models.

Post Keynesians argue that in principle, markets for final goods do not reach equilibrium between aggregate supply and demand by adjusting prices but by adjusting quantities of traded products, as the production of finished goods is elastic. These prices are "cost-determined". This statement does not apply to the prices of raw materials, such as oil, which are "demand-determined" as explained by Kalecki (1971, p. 11). In dynamic (Stock Flow Consistent) Post Keynesian models there is no general equilibrium in the neoclassical sense, but instead of that asymptotic trajectories may exist. These trajectories are often path dependant (there is hysteresis). Certain systems may display oscillatory or chaotic behaviour and in these cases the concept of long run equilibrium is not applicable. The future can be forecast in the short or medium run but is fundamentally unknown in the long run. It may be possible to extrapolate the current trends but this is not a meaningful forecast. Only a short run (dynamic) equilibrium between flows can be defined; stocks and stock-flow norms change as a result of accumulation processes (Godley & Lavoie, 2007). In the short run equilibrium is reached on product and labour markets because of quantity adjustments, price based adjustment processes only operate on financial markets. Kalecki (1971, p.165) wrote: *"In fact, the long run trend is but a slowly changing component of a chain of short run situations; it has no independent entity"*. This is consistent with the approach present in the modelling and simulation of physical, biological and technological systems and processes.

2.3 J. M. Keynes and the early controversies about Loanable Funds Theory and the Natural Rate of Interest

Special consideration needs to be given to one of the elements of neoclassical theory functionally related to the money neutrality axiom which is called the Loanable Funds Theory (LFT). There are three different versions of this theory: the classical, the Wicksellian and the neo-Wicksellian (introduced by New Keynesians in their models). It is easy to confuse them and apply the critique of the classical theory to the neo-Wicksellian version. Such a critique, usually based on the observations that bank money is endogenous, misses the point entirely. The neo-Wicksellian version of LFT is already based on the assumption that bank money is endogenous. The New Keynesian approach, integrating the short-run IS-LM analysis with the long-run neoclassical growth framework will be discussed later in Section 2.4.

2.3.1 The classical version of the LFT and its critique

Neoclassical synthesis models are based on the classical version of the interest rate theory, which would apply to a cash economy. It is assumed that a certain amount of money is withdrawn from circulation in each period by savers (not spent on consumption goods). This amount is then lent to investors on the loanable funds market. A lower volume of saving means a lower volume of investment. A lower interest rate means a lower inducement to save. The price of money that is the interest rate is determined as usual by the law of supply and demand. All the monetary tokens which have been withdrawn from circulation by savers are reintroduced into circulation (spent on investment goods) by investors. Money is neutral and only lubricates the process of exchange of real goods because the economy operates as if real capital was lent and borrowed. Banks and other financial institutions act as passive intermediaries between the savers and the borrowers (investors).

There is no need to introduce nominal variables into neoclassical growth models, due to the neutrality of money. Changes in the quantity of money in circulation only affect the price level, not the quantity of goods produced.

The introduction of a representative agent which simultaneously acts as a consumer, producer and saver-investor further reinforces the Say's law and money neutrality in neoclassical models of an economy, such as the Solow growth model (Garın, Lester & Sims, 2018).

Keynes (1936) rejected the classical theory of interest in his General Theory.

> *The classical theory of the rate of interest seems to suppose that, if the demand curve for capital shifts or if the curve relating the rate of interest to the amounts saved out of a given income shifts or if both these curves shift, the new rate of interest will be given by the point of intersection of the new positions of the two curves. But this is a nonsense theory. For the assumption that income is constant is inconsistent with the assumption that these two curves can shift independently of one another. If either of them shift, then, in general, income will change; with the result that the whole schematism based on the assumption of a given income breaks down. (Keynes, 1936 p.179)*

Keynes considered interest rate to be a monetary-only phenomenon (determined by liquidity preference of households if the central bank does not attempt to control it). The liquidity preference theory introduced in Chapter 13 is one of the centrepieces of the General Theory.

> *It should be obvious that the rate of interest cannot be a return to saving or waiting as such. For if a man hoards his savings in cash, he earns no interest, though he saves just as much as before. On the contrary, the mere definition of the rate of interest tells us in so many words that the rate of interest is the reward for parting with liquidity for a specified period. For*

the rate of interest is, in itself, nothing more than the inverse proportion between a sum of money and what can be obtained for parting with control over the money in exchange for a debt for a stated period of time. (Keynes, 1936 p.166-167)

Keynes (1936) identified the following motives for holding liquid assets:

- the transactions-motive (to finance current transactions)
- the precautionary-motive (to provide a cushion for unexpected future expenses because the future is uncertain)
- the speculative-motive (to allow for profiting from an increase in interest rates or falling stock prices)

Later in Keynes (1937), the following was added:

- the finance-motive (to finance planned investment)

Keynes thought that reducing the rate of interest would not starve the economy of investment funds but would actually stimulate investment.

The justification for a moderately high rate of interest has been found hitherto in the necessity of providing a sufficient inducement to save. But we have shown that the extent of effective saving is necessarily determined by the scale of investment and that the scale of investment is promoted by a low rate of interest, provided that we do not attempt to stimulate it in this way beyond the point which corresponds to full employment. Thus it is to our best advantage to reduce the rate of interest to that point relatively to the schedule of the marginal efficiency of capital at which there is full employment. (Keynes, 1936 p.375)

Keynes did not reject the marginalist approach in economics. His analysis was using the term “marginal efficiency of capital”. Keynes (1936, p.144) wanted to *“show that the succession of Boom and Slump can be described and analysed in terms of the fluctuations of the marginal efficiency of capital relatively to the rate of interest.”*.

Marginal efficiency of capital is an expectational parameter as entrepreneurs can only anticipate how much their investment will earn in the future. Changes in business confidence manifest themselves as changes of the expected marginal efficiency of capital.

2.3.2 Critique of the Wicksellian version of the LFT

According to Bertocco (2009), the key idea the of Wicksell's version of LFT was while the classical Quantity Theory of Money (QTM) introduced by Ricardo held in the cash-only monetary system, an analogous concept needed to be introduced for the system with bank money. In a classical (cash-only) system money holdings of agents would be proportional to the amount of money intended to be spent during the period of time. In this kind of economy it is possible to control the level of nominal prices by exogenously changing the quantity of money in circulation. The introduction of bank money makes the quantity of money in circulation an endogenous variable. The level of prices no longer depends on the quantity of money. For Wicksell it depends on the price for which money can be borrowed from the banking system (that is on the interest rate). Wicksell wanted to rescue the QTM by introducing a concept of a "natural" rate of interest which is a reference non-observable rate at which capital would be lent in kind. The natural rate depends on supply and demand. This rate of interest would exist in the economy without (endogenous) bank money, where money is neutral (a Keynesian "real-exchange" economy). In a world with bank money the natural rate of interest is a non-observable parameter while the monetary rate of interest is a separate and observable variable. The capital market and money market also exist separately. Wicksell claims that only if the monetary rate of interest corresponds to the natural rate, will prices be stable.

> *If, other things remaining the same, the leading banks of the world were to lower their rate of interest, say 1 per cent. below its ordinary level, and keep it so for some years, then the prices of all commodities would rise and rise and rise without any limit whatever; on the contrary, if the leading banks were to raise their rate of interest, say 1 per cent. above its normal level,*

and keep it so for some years, then all prices would fall and fall and fall without any limit except Zero. (Wicksell, 1907, p. 213).

The monetary rate of interest usually approaches the natural rate of interest. The presence of endogenous bank money can then be ignored since the economy behaves as if it was a real-exchange economy. Money neutrality has been restored in the long run.

For Keynes, multiple Wicksellian "natural rates" could exist, corresponding to different rates of employment. A "neutral rate of interest" would be a special case of a Wicksellian "natural rate", corresponding to full employment.

If there is any such rate of interest, which is unique and significant, it must be the rate which we might term the neutral rate of interest, namely, the natural rate in the above sense which is consistent with full employment, given the other parameters of the system; though this rate might be better described, perhaps, as the optimum rate. The neutral rate of interest can be more strictly defined as the rate of interest which prevails in equilibrium when output and employment are such that the elasticity of employment as a whole is zero. (Keynes, 1936 p.243)

Keynes (1936 p.316) explained a crisis in a trade cycle by *"the collapse in the marginal efficiency of capital"*. He stated that *"the collapse in the marginal efficiency of capital may be so complete that no practicable reduction in the rate of interest will be enough."*

According to Levrero (2019), the view that the schedule of marginal efficiency of capital and the interest rate are the parameters predominantly determining the volume of investment exposed Keynesian theory to a later "reconciliation" with the LFT. If lowering the interest rate causes a "mechanical" increase in investment resulting in growth of the employment and full employment is achieved when the interest rate is equal to its "neutral" value, all that is needed to restore the LFT in the Wicksellian form is demonstrating that lower unemployment leads to higher inflation (the existence of a stable Philips curve) and that the "neutral" rate is determined by "productivity and thrift". The critique of these views is provided in Section 2.4. Investment decisions of the majority of individual companies do not depend strongly on the interest rate unless this rate exceeds the threshold known as the "hurdle" rate. This is due to the uncertainly in estimating the rate of return on newly purchased capital goods. The magnitude of this uncertainty far exceeds the differential of estimated marginal efficiency of capital and expected long-run rate of interest. Instead of this, in the long run companies try adjusting investment so that their rate of capacity utilisation approaches what is called the normal or target rate (Nikiforos, 2018).

Keynes (1936 p.386) expressed negative views about the class of rentiers while supporting entrepreneurs in the tradition of classical liberalism. He thought that keeping interest rates low enough to ensure full employment would also lead to *"the euthanasia of the rentier, and, consequently, the euthanasia of the cumulative oppressive power of the capitalist to exploit the scarcity-value of capital".*

Bertocco (2009) provided a detailed analysis of Keynes's (1937) views on Wicksellian version of the Loanable Funds Theory included in the responses to the critique of his General Theory written by Ohlin (1937). Ohlin's critique appears to be quite confusing but it was precisely this confusion that allowed for the resurrection of Loanable Funds Theory first in the neoclassical and later in the New Keynesian synthesis. An attempt to reconcile the classical theory of interest with the Keynesian theory of effective demand led to what was called by Joan Robertson a "bastardised Keynesianism". For Ohlin *"the rate of interest is simply the price of credit, and that it is therefore governed by the supply of and demand for credit"* (Ohlin, 1937 p.221). This statement was a step back from what Wicksell had written in 1898 – that in an economy with bank money *"supply and demand of money have in short now become one and the same thing"* (as cited in Bertocco, 2009, p.4). Ohlin accepted the Keynesian arguments against the classical theory of interest as not applicable to the monetary economy but this did not mean that he was willing to accept the idea of "euthanasia of the rentier" or rather of the saver. In Ohlin's world savers still affect the interest rate – their willingness to accept and hold as assets someone else's liabilities determines the supply of loanable funds.

People's willingness to hold the different claims and other kinds of assets every day governs the supply of credit. The total supply of claims, etc., governs the demand for credit. In each market for the different claims, etc., supply and demand are made equal by price. These prices for interest-bearing claims on certain fixed sums determine the rates of interest. It is quite obvious that this reasoning in gross terms leads to the same result as the net analysis above. (Ohlin, 1937 p.224-225)

Keynes did not agree with the views that saving has an impact on interest rates or that it is a necessary source of funds needed by entrepreneurs to invest.

For "finance" is essentially a revolving fund. It employs no savings. It is, for the community as a whole, only a bookkeeping transaction. As soon as it is "used" in the sense of being expended, the lack of liquidity is automatically - made good and the readiness to become temporarily unliquid is available to be used over again. Finance covering the interregnum is, to use a phrase employed by bankers in a more limited context, necessarily "self-liquidating" for the community taken as a whole at the end of the interim period. (Keynes 1937, p. 666)

2.4 The New Keynesian, neo-Wicksellian version of Loanable Funds Theory

While classical Loanable Funds Theory is still taught to undergraduate students and used in everyday economic debates, New Keynesian economics is based on the neo-Wicksellian version of the theory, which has been developed by Woodford (2003) as a modern extension of the ideas discussed in Section 2.3.2. This version of LFT applies to a New Keynesian model of the economy built using the "microfoundations". It is assumed that a single representative rational agent performs intertemporal maximisation of expected value of a discounted sum of period contributions to utility.

The key concept of the modern version of LFT is the idea of Natural Rate of Interest (NRI) which corresponds to Keynesian “neutral rate” in regards to aggregate demand in the short run. This rate is no longer determined by ex-ante equilibrium on the market for loanable funds as investment and saving are equal ex-post in a monetary system based on credit money. The NRI is an unobservable parameter determined by microeconomic parameters such as the household utility discount factor, the rate of economic growth and the elasticity of intertemporal substitution (Brand, Bielecki & Penalver, 2018). If no market imperfections are included in the model, the rate of return on capital coincides with the NRI. The number of parameters affecting the value of interest rate increases if transaction frictions are introduced to the model. Stochastic shocks can affect current value of NRI. If the actual rate of interest differs from the NRI, inflation is not constant and unemployment differs from its natural rate. The output gap, the inflation and unemployment rates are described by New Keynesian Phillips curve. Due to market imperfections the Natural Rate of Unemployment is assumed to be greater than zero. In order to provide price stability central banks have to follow monetary rules which is a task similar to tracking the unobservable NRI. There is a trade-off between output and inflation in the short run but not in the long run. If the NRI becomes negative and standard monetary policy is not capable of closing the output gap, fiscal policy may work. This temporary situation is called a “liquidity trap”. However in the long run households are Ricardian and fiscal policy loses traction due to diminishing spending multipliers. If the rate of interest is closely tracking the NRI then the financial system is assumed to be close to equilibrium in regards to the market for capital goods. The economy behaves as if there were no frictions and due to “divine coincidence” there is no output gap and inflation is stable. The principle of operation of the monetary system is then the same as described by the classical LFT, where capital is lent “in kind” on the market and the cost of borrowing is determined by the marginal rate of return on capital. Money is neutral in the long run and Say’s law applies. If the government is to increase spending, it has to simultaneously increase taxation by the same value as if spending was financed ex-ante.

The use of both classical and neo-Wicksellian versions of LFT in public debates could be the root cause of confusion between New Keynesians and Post Keynesians. Macroeconomic textbooks written by prominent New Keynesians explain the functioning of banks as a process of intermediation between savers and borrowers. An example of which is the popular undergraduate textbook, "Macroeconomics" written by Krugman and Wells (2015), where a whole section (pp. 722-731) describes the functioning of the classical market for loanable funds. Ex-ante savings or at least bank reserves are required to issue loans. Governments have to tax or borrow in order to spend (government budget constraint is presented as an ex-ante condition even in the short-run). It is quite likely that intuitive understanding of how the economy works in the long run is still deeply rooted in the classical LFT and exceptions in the New Keynesian theory are only made for short-run analysis.

Post Keynesians argue that bank money is endogenous and that there is no loanable funds market determining interest rate and since the currency is fiat money, the government budget constraint is an ex-post accounting identity (Syll, 2015). When Post Keynesians criticise New Keynesian theory for including elements borrowed from the classical LFT, this critique can be promptly deflected by stating that New Keynesian theory is fully integrated with modern understanding of the financial system. It is often claimed that Post Keynesian theory lacks sophisticated model based on microfoundations and that active fiscal policy was discredited in the 1970s because it led to accelerating inflation.

Levrero (2019) has presented a critique of neo-Wicksellian concept of Natural Rate of Interest (NRI) and New Keynesian models in general. He claims that:

- There is no clear and reliable econometric method of estimating NRI from real economic data. Multiple methods were discussed but none of them delivers statistically certain results. Model-based estimates may produce results influenced by the design of the structural model.

- The relations between prices, output and the rate of interest may differ from what has been specified in New Keynesian models. Econometric studies demonstrate that effects of changes of the interest rates on consumption and investment are uncertain and vary according to circumstances. Investment decisions depend mostly on expected changes in aggregate demand. Interest rate elasticity of output appears to be low and asymmetric. This means that the whole New Keynesian model based on neo-Wicksellian LFT in the long run and using the difference between the current interest rate and NRI to determine aggregate demand in the short run is invalid.

- Consumption of low-income households mostly depends on past and current income, these households do not make spending decisions based on intertemporal optimisation of a utility function.

- Investment may also not increase when the interest rates fall. While some components of investment such as investment in private residential real estate may increase, investment in fixed productive capital is mostly driven by expected changes in aggregate demand not by changes in interest rates.

- A fall in the interest rates does not cause higher inflation, a phenomenon known as the Gibson paradox. It can be explained in the context of cost pricing. If the markup remains constant, a reduction in the interests costs will cause the final price of the product to fall. This would manifest itself as a temporary drop in the rate of inflation after a reduction in the interest rates.

- Sraffa (1975) has demonstrated that the value of capital (consisting of multiple different capital goods) depends on prices, which in turn depend on the rate of interest. It is impossible to formally derive a decreasing demand schedule of investment as a function of rising interest rates. For a multi-commodity economy an aggregate production function cannot be derived due to re-switching and reverse capital deepening. Multiple equilibria with different volumes of investment flow for a given interest rate may exist. Assuming that the rest of the New Keynesian theory is valid and can be used to describe the real economy, this means that multiple values of NRI may exist.

- The actual transmission channel of monetary policy may be via the income distribution channel.
- Changes in the rate of interest affect currency exchange rates and may have an impact on prices and the rate of inflation by changing nominal prices of imported commodities and also the demand for exported products.

The view that the (variable) business-cycle component of the investment to GDP ratio does not primarily depend on the changes in interest rates has been confirmed by Andrle, Brůha and Solmaz (2017) in their econometric study based on Dynamic Principal Component Analysis. This methodology allows for the identification of dominant sources of co-movements of other variables without making prior assumptions about the structure of the model. The dominant component is the aggregate demand, not monetary policy settings.

The econometric studies mentioned by Levrero (2019) reveal a low sensitivity of corporate investment to the interest rate. This has also been confirmed by Sharpe and Suarez (2015) who analysed data from surveys of CFOs of companies, The majority of surveyed firms would not alter their investment plans if the interest rate changes by one or two percentage points. Only if the interest rate exceeds a specified hurdle value (which is greater than 10%), will firms reduce their investment spending.

Let us assume that the aggregate volume of investment planned for the next period is a deterministic function of several variables (including the interest rate). As long as an even very small increase in investment can be observed when the interest rate falls (the IS curve is downward sloping), advocates may claim that the New Keynesian model is a valid representation of reality and that the NRI exists. However this reasoning is flawed, unless the differences between the actual rate of interest and the NRI are the dominant source of changes in the aggregate investment (which affects the GDP due to the usual multiplier effect). If we assume that firms try to maintain constant capacity utilisation ratio in the medium run, any temporary increases in investment caused by an expansionary monetary policy, which are subsequently not validated by the increase in the expected sales due to forthcoming increase in the aggregate demand, will simply result in a fall in the investment in the next period. This would seemingly correspond to the NRI falling as a result of the fall in the actual rate of interest. If the sensitivity of the planned investment to changes of the interest rate is low and other parameters (such as the aggregate demand, affected by exogenous components) dominate then the concept of the NRI as an unobservable structural parameter determining the behaviour of the economy is meaningless as its value is unstable and depends on these parameters. As already mentioned in Section 2.3.2, it is the uncertainty facing individual companies and investors about the future rate of return on investment (Keynes, 1936), that makes investment decisions quite insensitive to changes in the interest rate.

All versions of the Loanable Funds Theory should be rejected in general as not applicable to the modern monetary economy. The Post Keynesian alternative to New Keynesian modelling framework (the SFC framework) is described in Chapter 3. This framework will be used to develop a model based on the idea of Sraffian Supermultiplier. The model is described in Chapter 4.

2.5 The Macroeconomics of the Great Depression and the Global Financial Crisis – a New Keynesian approach

Understanding the economic mechanism leading to periodic recessions has been one of the main goals of macroeconomics since it emerged as a distinct branch of economics in the 1930s, after the publication of the "General Theory" by Keynes (1936).

A concise analysis of the macroeconomic mechanism of the Great Depression can be found in the first chapter of Bernanke (2000). The root causes of the depression were monetary shocks in the US economy which spread to the real economy and were transmitted abroad. The demand-side of the economy was affected by a deflation-induced financial crisis. The supply-side of the economy experienced an increase in real wages as nominal wages failed to adjust quickly enough to falling nominal prices. Bernanke argued that the mechanism of the functioning of the international gold standard which was in place in the late 1920s and early 1930s contributed significantly to the depth and persistence of the economic downturn. The countries that left the gold standard recovered quicker because they were able to reflate the money supply.

Bernanke acknowledged that in the short run, money was not neutral. According to him, borrowing and debt repayment merely transfers spending power from one group of economic agents to another, but agency costs on capital markets may rise if asset prices fall and the efficiency of financial intermediation on the part of the banks is compromised. Analysing panel data from several countries, he only found weak statistical evidence for a negative impact of wage stickiness on economic recovery. He tried instead to explain the slow recovery on inappropriate government intervention and monopolistic behaviour on the part of some corporations.

In his Testimony before the Congressional Financial Crisis Inquiry Commission, delineating the causes of the Global Financial Crisis, Bernanke (2010) distinguished between triggers and vulnerabilities. The main trigger was the prospect of losses on residential mortgage loans to subprime borrowers, when house prices started to decline. But these losses weren't large enough on their own to cripple the global financial system. Other vulnerabilities contributed to the scale of the downturn. In mid-2007, financial institutions started experiencing problems rolling over asset-backed commercial papers. The short-term interbank loans market also became congested. In June 2007 there was a "sudden stop" in syndicated lending to large and risky corporate borrowers, caused by falling confidence of short-term investors. These syndicated loans were issued by "special purpose vehicles", issuing collateralised loan obligations. Bernanke attributed some of the vulnerabilities of the financial sector to the growth of a shadow banking system, reliant on short-term uninsured funds (such as repos and commercial paper). This made the system prone to bank runs. Initially the Federal Reserve could not inject liquidity onto this market because lenders were not regular banks. In Europe, the shadow banking system experienced problems due to the lack of access to dollar-denominated funds. The Fed had to establish dollar liquidity swap agreements.

Bernanke highlighted the deteriorating quality of risk management prior to the crisis as one of the factors increasing the level of vulnerability of the system. Securitisation of mortgage loans hid the risk. Many households and financial institutions borrowed more than they could service and were left with negative equity when house prices started falling. This led to mass defaults and further decline in house prices. Bernanke listed several gaps in the statutory framework of financial regulations prior to the crisis. There was no government entity capable of managing systemic risks. Government-sponsored financial institutions involved in housing, Fannie Mae and Freddie Mac, were not properly supervised. Some of the financial institutions were "too-big-to-fail". In the end it was congressional action to recapitalise the banking system that restored financial stability. Bernanke mentioned the possibility that low interest rates could have contributed to the growth of the housing bubble but he was not convinced that this had been a significant factor.

John Taylor (2009) was more critical of the policies of the Federal Reserve than its Chairman. He noticed that in the lead-up to the GFC (2002-2006) the Fed had set the interest rate much (up to 3%) below what the Taylor rule had suggested. This was a discretionary intervention undertaken after the collapse of the dotcom bubble (2001) in order to stave off deflation which had affected Japan in the 1990s. Taylor argued that following the actual interest rate trajectory deviating from the monetary rule could have possibly brought about a housing boom and bust. He rejected the explanation that low interest rates were caused by a global excess of saving (as the equality of global real saving and investment is an ex-post global accounting identity). Taylor argued that the impact of housing bubbles was the most severe in these countries which deviated the most from the Taylor rule. In the US the use of sub-prime mortgages and securitisation increased and hid the risk. After the financial crisis erupted, the rise of Libor-OIS (overnight index swap) spread (up to 3.5%) affected the economy by increasing the costs of lending. The higher levels of spreads especially the Libor-Repo (unsecured-secured) spread could have been explained by an elevated level of counterparty risk not the lack of liquidity on the interbank market. According to Taylor, the direct lending of reserve funds to banks by the Fed did very little to calm the market as the root cause of the crisis had been misdiagnosed. Taylor was also critical of the temporary cash infusions which were handed over to households in 2008 and the initial cuts to interest rates which in his view only fuelled the increases in oil prices, which exacerbated and prolonged the crisis in the real economy. He criticised the lack of predictability about Treasury-Fed interventions what increased the level of uncertainty experienced by financial markets. Taylor (2010) was even more critical of the discretionary counter-cyclical fiscal actions (claiming that they had had no positive impact on the level of personal consumption) and rescue programs helping financial firms. His recommendation was to continue the macroeconomic policies from the period before the dot-com crisis (the Great Moderation) such as reducing the budget deficit and implementing a rules-based monetary policy, targeting low inflation and stable economic growth. Taylor recommended not using unconventional tools such as quantitative easing.

Lawrence Summers (2014) admitted that after the GFC the American economy has been experiencing a sluggish growth (the "secular stagnation"). The potential growth of the economy was revised downwards as a result of the hysteresis, mainly because of the fall in capital investment and the reduction in labour input (slower productivity growth was the third and less important factor). Summers attributed some of the GDP growth of the period 2002-2007 to the effects of an unsustainable housing bubble. Before the 2001 crisis the economy had been stimulated by a stock market bubble. Summers blamed the apparent economic stagnation on the decline of the equilibrium real rate of interest (NRI) due to the shift between saving and investment (this explanation is based on the Loanable Funds Theory framework). The decline in the NRI would prevent achieving full employment because of the zero lower bound on nominal interest rates. Low nominal interest rates supported risk-seeking behaviour of investors which diminished the stability of the economic system. As a policy recommendation, Summers supported keeping the interest rates low and using fiscal policy to increase the demand, because fiscal multipliers are high when the natural rate of interest falls below the zero bound.

Despite being expressed in the language of Loanable Funds Theory, the explanation of the crisis and stagnation provided by Lawrence Summers is actually quite consistent with the views expressed by many Post Keynesians such as Blecker (2016). Changes in the parameters of the consumption function moved the short-term equilibrium of the economy (operating without an external stimulus) towards lower utilisation of labour and productive capacities. These changes have been explained by Post Keynesians by redistribution of national income towards higher-income households. Debt-financed spending (initially corporate capital investment during the dot-com bubble, then housing construction during the real estate bubble) stimulated the economy but these processes were unsustainable. The secular stagnation corresponds to the "equilibrium" of the economic system without external stimulation. The long-run growth trajectory of the economy is affected by low accumulation of productive capital.

Benigno and Fornaro (2017) have linked low productivity growth after the GFC with the lower aggregate demand and lower firm profits. This has negatively affected the level of investment in innovation. By making productivity growth endogenous, the authors argue that the economy may enter a stagnation trap with a negative NRI. While the model is New Keynesian, the causal link between low aggregate demand, low investment in R&D and low productivity growth is a plausible explanation of the fall in productivity growth depicted in Figure 5.

2.6 The theoretical debate about the GFC and Secular Stagnation

Neoclassical and New Keynesian models were not successful in explaining several important macroeconomic phenomena related to the GFC and the sluggish recovery after the crisis.

- Fiscal multipliers applied when changes in interest rates could not offset fiscal tightening were as a result significantly underestimated in DSGE models – they were about 1.5 instead of 0.5 (Blanchard & Leigh, 2014)
- As Stiglitz (2018, p.71) claimed *"The DSGE models fail in explaining these major downturns, including the source of the perturbation in the economy which*

gives rise to them; why shocks, which the system (in these models) should have been able to absorb, get amplified with such serious consequences; and why they persist, i.e. why the economy does not quickly return to full employment, as one would expect to occur in an equilibrium model. These are not minor failings, but rather go to the root of the deficiencies in the model."

- The prolonged downturn experienced by Eurozone economies after the fiscal consolidation in 2012 was not forecast (Rannenberg, Schoder & Strasky, 2015)
- The absence of price and wage deflation during periods of sustained high unemployment has not been explained (Krugman, 2018)

Post Keynesian models assume that economic activity is usually constrained by the level of demand. Since not all the disposable income may be spent (consumed or invested), the system reaches a short-run equilibrium below the full employment (see Section 4.1.2).

Post Keynesian models yield realistic values of fiscal multipliers. The value of the spending multiplier in the long-run is not significantly lower than in the short-run (in fact it may even be higher, see Section 4.16.3). This fully explains why "expansionary fiscal contraction" could have only resulted in a recession, despite being offset by the European Central Bank through a drop of the interest rate from 1% to 0%.

Post Keynesian inflation theory is based on the conflicting-claims model, where workers (trade unions) and corporations use their bargaining power and the ability to set prices in attempt to achieve what is considered by each side to be a fair share of income. Both demand-pull and cost-push inflation can be explained in this context as processes emerging either due to excessive aggregate demand or an increase in the costs of production. The absence of price and wage deflation during the GFC can be explained by the presence of a middle flat segment in the Post Keynesian real wage target schedule as depicted on Figure 8.6 in Lavoie (2014).

The level of aggregate demand is only indirectly affected by the development of the supply side, specifically by the technical progress. The development of technology and productivity growth depends on investment which is demand-driven. This creates a positive feedback loop in the long-run as the availability of new, more technically advanced products creates demand (“markets”) for these products (Mazzucato, 2018). But the aggregate demand also strongly depends on income distribution which is discussed in Sections 2.7 and 4.1.2.

There has been a further debate among heterodox economists over whether sources of instability in the capitalist economy, manifesting itself as the business cycle, are endogenous within the productive sector (related to investment decisions) or exogenous (originating in the household and foreign sectors). Both productive and household sectors can finance long-term investment by taking loans. If the stock of debt to revenue or income ratio becomes significant, the boom-bust cycle identified by Minsky (1975; 1992) in his financial instability hypothesis may develop.

While the instability was initially linked with the rising financial fragility within the corporate sector, the magnitude of leveraged speculation in residential real estate assets in the early 2000s was much larger. The consequences of rising indebtedness of households and the effects of bursting of the housing bubble were also much more severe than the effects of bursting of the stock market bubble. The recession of 2001 was relatively mild while the GFC was the most severe downturn since the Great Depression (Figure 1).

Systemic financial fragility was analysed in a stock-flow consistent framework by Passarella (2011; 2012). Dos Santos and Macedo e Silva (2009) linked financialisation, income distribution changes and Minskyan financial fragility.

The Minskyan perspective allows to identify the processes leading to the emergence of a boom-burst cycle involving leveraged speculation on rising prices of assets which is linked with the increased financial fragility. The cycle ends with capital losses, defaults and disruption to the functioning of the financial sector which may spread to the wider economy. Looking from the point of view of the whole economy, the cycle is endogenous (it develops within the national economy, it is not a result of external factors, such as exogenous productivity shocks). The cycle involving leveraged household investment in real estate can be considered as exogenous from the point of view of the corporate sector and the majority of households not involved in the speculative activities, which only experience significant fluctuations in the aggregate demand, disposable income, expected wealth position and suffer from possible side effects of the defaults in the banking sector. This approach is consistent with the idea of the Sraffian Supermultiplier, discussed in Section 2.8.

2.7 Secular Stagnation as a result of rising income inequality

This section contains the Post Keynesian analysis of the growth slowdown experienced after the GFC.

According to Bhaduri and Marglin (1990) a demand-driven economy can operate in two regimes – the stagnationist and the exhilarationist. In the stagnationist regime an increase in the wage share would lead to an increase in the GDP because of the higher marginal spending propensity of workers. In the exhilarationist regime a higher share of profits would lead to an increase in investment, driving the GDP higher due to the multiplier effect.

Cynamon and Fazzari (2015) provided econometric evidence suggesting that changes in the income distribution between various income groups (rising income inequality) contributes to weakening of the aggregate demand in the long run. This is consistent with the observation that the American economy operates in the stagnationist regime.

Stiglitz (2015) strongly supports the view that inequality harms the economy due to the weakening of aggregate demand. This had encouraged loose monetary policy which in turn led to a housing bubble. In his opinion, rising inequality also causes more indirect effects such as lack of educational opportunity, negatively affecting productivity growth in the long run.

2.8 The role of autonomous expenditures in the Sraffian Supermultiplier as the main driving force of the business cycle

In order to explain the business cycle, some economists would like to arbitrary separate the parts of the economy which passively react to the changes from the components which actually generate shocks. Empirical evidence supports the view that it is semi-autonomous household expenditures financed by borrowing, rather than endogenous changes in the rate of profit (as in the Goodwin model) which drive the business cycle (Fiebiger, 2017). The transmission channel from the monetary to the real sphere of the economy is related to the capacity for borrowing and debt repayment to both create and destroy spending power, as argued by Kalecki (1971). The Sraffian Supermultiplier (SSM) framework (based on the original work of Serrano) has been presented in Serrano and Freitas (2017). The explanation of the business cycle as a process mainly driven by semi-autonomous investment in real estate has been developed by Fiebiger and Lavoie (2017).

Nikiforos (2018, p.4) explains the behaviour of the Sraffian Supermultiplier model in the short run in the following way.

> *At the heart of the SSM model lies autonomous, non-capacity-generating spending. Autonomous means that is not affected by other economic variables within the system. These kinds of expenditures include debt-financed consumption, capitalist consumption, residential investment, government expenditure, and exports. Since the system, by assumption, converges to a balanced growth path with a normal rate of utilization, autonomous expenditure—whose growth is also assumed to be exogenous—sets the tone in the long run and the whole system grows at the rate of growth of the autonomous expenditure. Changes in autonomous spending are transmitted to output through the supermultiplier. Since the growth rate of output is pinned down by the exogenous growth rate of autonomous*

spending, changes in other variables that usually have a more central role in demand-driven models (e.g., saving rate, income distribution) have only a transitory or level effect on economic activity.

In these models, the long-run capacity utilisation adjusts to its desired level due to changes in investment. The accumulation rate converges to the growth rate of autonomous demand. Autonomous demand (per person) has to follow the long run rate of productivity growth, otherwise stock-flow ratios of debt to GDP will not stabilise. Thus, autonomous demand is not truly "exogenous" and "autonomous". According to Nikiforos (2018) this inconsistency limits the validity of the model. Yet in the medium-run the private debt to GDP ratio has changed significantly in the economies around the world. This finding is more consistent with the Minsky's financial instability hypothesis than with the "static" interpretation of the SSM.

This apparent puzzle can be solved by realising that the trajectory of economic growth in the long run is not a response of an asymptotically stable dynamical system to one of several shocks. Multiple parameters describing the behaviour of the "passive" component of the system also significantly change in time. The growth in aggregate demand, including its semi-autonomous components cannot be separated from the growth in productivity and population, as already mentioned in Section 2.6. The model describes the behaviour of a closed feedback loop but it can still be claimed that the parameter which drives the changes is the aggregate demand (as argued by Andrle et al., 2017).

Chapter 3 Choosing the modelling methodology

3.1 The limitations of macroeconomic modelling

Bronk (2011) analyses epistemological difficulties with neoclassical micro-foundations and explores their impact on how neoclassical macroeconomic theory can be used to describe and predict actual economic phenomena. The validity of economic theories ("positive" economics) is supposed to be determined by the accuracy of predictions made by using these theories. The process of describing and explaining the reality in order to predict the future involves creating scientific models. Different categories of models are described in Section 4.1.1.

Bronk (2011, p.13) provides a critique of the neoclassical and New Keynesian approach towards using economic models to predict a fundamentally uncertain future. If these objective limitations are acknowledged and accepted, the role of models in the scientific process should be different, similar to what is assumed in empirical sciences.

> *If economics is to be used to explain what is going on in complex systems and innovative markets, it will have to accept different standards of proof than the rigorous testing of ex ante spot predictions (Bronk, 2009, 27). Indeed, the very failure of most predictions in the area of macroeconomics and finance theory over the last decade should itself prompt questions and render attractive this downgrading of the status of apparently precise predictive models. As Hodgson (2011b, 191) puts it, the 'underlying error lies in overestimating the importance and possibility of prediction.' Nor does economics need to be embarrassed about any loss of scientific credentials if it relies on models that explain and simulate rather than predict precise outcomes. After all the queen of modern natural sciences – biology – rarely attempts to predict the future with any precision, recognising the central importance of random mutations, threshold effects and increasing returns. And there are many who would argue that the time has come for economics to take more inspiration from biological*

metaphors, and 'follow biology in embracing complexity and downgrading prediction in favour of the primary goal of causal explanation' (Ibid., 192).

3.2 Comparing the SFC methodology against the DSGE

Two main categories of modern economic models could be considered as suitable for analysing dynamic processes such as the Global Financial Crisis – DSGE and SFC.

A critique of the DSGE modelling approach has been presented in Caiani et al. (2016) in the context of attempting to build a simple agent-based stock-flow consistent benchmark model. The authors mentioned the following issues:

- microeconomic and macroeconomic models operate at different levels of abstraction, this may lead to a fallacy of composition; DSGE models assume the existence of representative agents
- real economic agents have limited access to information and they are not fully rational,
- the financial system is usually oversimplified in DSGE models and its interaction with the real economy is difficult to model

Stiglitz (2015) highlighted the impact of income inequality on the economic growth in the medium-run. Multiple social classes can be introduced to DSGE models but then these models will have to rely on ad hoc assumptions which are inconsistent with the microfoundations describing the behaviour of a representative agent maximising its utility in an infinite time frame.

DSGE models rely on a form of aggregate production function with constant elasticity of substitution, usually a Cobb-Douglas function. These functions has been inherited from neoclassical growth models. The critique of Cobb-Douglas production function has been provided by Shaikh (1974) in the context of Cambridge capital controversies. It does not describe the flow of real output as a function of the stock of heterogeneous physical capital used for production and flow of labour supplied by workers but rather links the flow of nominal profits and wages with the flow of nominal output. Shaikh has demonstrated that marginal productivity of labour and capital in a Cobb-Douglas production function are artefacts of distribution of the gross revenue between wages and profits. If the distribution of gross revenue changes because of the monopoly or monopsony on the labour market then the coefficients of the production function should also be updated otherwise the model is algebraically inconsistent.

As pointed by Nikiforos and Zezza (2017) the Stock-Flow Consistent approach is based on several unique principles not present in neoclassical (DSGE) models. The models are supposed to realistically describe the economy at the macro level. The following are the modelling principles:

- A model is set up as a system of linked balance sheets and transaction-flow matrices, built on the principle of quadruple entry. Accounting consistency is preserved on flow, stock and stock-flow levels. These ex-post identities provide the structural foundations of the model.
- A Keynesian-Kaleckian demand-led closure determines causality (past and current investment and consumption decisions determine the current flow of profits and as a consequence, saving). This is expressed in the form of ex-ante behavioural equations.
- Corresponding nominal and real variables are linked. The interaction between the financial and real spheres of the economy is accounted for in an integrated way.
- Models employ "empirically plausible behavioural rules" (Barwell & Burrows, 2011, p. 8) instead of neoclassical microfoundations.
- Short run equilibrium is reached through sales-production volume and asset price adjustments – equilibrium is not a state of rest but merely a state in which supply meets demand on various markets.
- The system traverses through time by integrating flows into stocks and updating expectational variables. The long-term equilibrium might be defined as a trend or asymptotic trajectory and may include economic growth. In the long run stock-flow norms (Godley & Lavoie, 2007) could be stable but it is also possible to simulate an endogenous cycle or even deterministic chaos.

- Exogenous deterministic or stochastic signals can be applied to the model to investigate trajectories corresponding to various possible scenarios, exogenous behavioural parameters of the model can change in time.

While DSGE models are in the long run “supply-side driven”, SFC models are predominantly “demand-side driven” so unemployment is not a result of labour market frictions and exogenous shocks but rather inadequate aggregate demand. SFC models are suitable for modelling multiple social classes and distributional conflicts. DSGE models do not describe capitalism but rather market socialism as the optimisation problem is solved from the point of view a representative agent who both supplies labour and owns the means of production.

3.3 The origins of the SFC methodology

It can be claimed that SFC methodology has also evolved from what Turnovsky (1997) called “Traditional Macrodynamics”. Early econometric models (Tinbergen, 1940) were based on statistical data capturing well-defined macroeconomic stock and flow variables such as volumes of sales, investment, wages, profits and stocks of fixed capital, financial assets, etc.

An example of a dynamic macroeconomic model of the US economy (implemented in continuous time) can be found in Gandolfo (1993, Chapter 8). This model uses adaptive expectations in a simple Keynesian consumption function with the constant average propensity to consume applied to real net disposable income. The firms sector is described by a Constant Elasticity of Substitution (CES) aggregate production function including oil as one of the inputs, together with labour and capital. This approach to modelling the corporate sector needs to be rejected in the context of Cambridge capital controversy and objections raised by Shaikh (1974).

While mainstream economists embraced the “rational expectation revolution”, integrating the representative-agent microfoundations into the models, some Post Keynesian economists purged traditional “Keynesian” macroeconomic models from the early 1970s from the elements inspired by the Loanable Funds Theory, Say's law, the Cobb-Douglas or generalised CES production function and other artefacts of neoclassical economics. The economy is seen as a collection of linked balance sheets of individual sectors and all the transactions are integrated using quadruple accounting rules. This framework enforces rigorous stock-flow consistency which may be missing in other modelling frameworks.

3.4 The features of Stock Flow Consistent models

Stock Flow Consistent models belong to the wider category of Post Keynesian models. They are:

- deterministic (not stochastic),
- quantitative (not qualitative),
- dynamic (not static),
- combining the short-run and long-run perspectives,
- macroeconomic (not microeconomic or multi-agent),
- mathematical (not visual),
- often simulated (not evaluated analytically)
- often theoretical (but could be empirical).

If a dynamic model is simple and the statistical data not too noisy, it may be possible to estimate the values of the parameters from the calibration data, using one of the error minimisation methods. The majority of SFC models cannot be calibrated this way mainly due to their complexity. They have too many parameters, high quality calibration data is usually not be available and the goal of the simulation is only to demonstrate the emergence of certain macroeconomic phenomena rather than to build a forecasting tool. These models only aim at reproducing certain "stylised facts". The models may be calibrated by adjusting values of constant parameters and checking whether the trajectory resembles the statistical data. Initial values of state variables and estimated values of constant parameters are often taken from empirical (econometric) studies as in Burgess et al. (2016).

The following gaps and deficiencies of General Equilibrium models can be addressed by the SFC methodology:

- Fiscal multipliers are consistent with the observations because of the realistic aggregate consumption function (Godley & Lavoie, 2007, Chapter 3.4.2).
- The amplification of demand shocks and hysteresis are easy to model because of the high value of spending multipliers and the path-dependant effects of integration of the flow of net investment into the stock of productive capital.
- Since the models are demand-driven and have limited or no tendency to self-adjust towards full employment and full utilisation of productive capacities, persistent effects of fiscal withdrawal can be demonstrated (Godley & Lavoie, 2007 p. 162)
- The dynamic equilibrium of supply and demand is reached mainly by adjustment of traded volumes not by adjustment of prices of commodities (firms target constant markups over labour costs not adjust prices to marginal costs) which makes prices more sticky than in in GE models (Godley & Lavoie, 2007, Chapters 1.1 and 8.3.2)
- It is possible to build a realistic model of the production process, separating real production function and pricing equations.

3.5 The limitations of the SFC methodology

The Dynamic Stock-Flow Consistent modelling methodology also has its limitations (Caverzasi & Godin, 2014). The Keynesian critique of early econometric models (Keynes, 1939) might apply to SFC models as there is no guarantee that macroeconomic parameters such as the coefficients describing the consumption function, will remain constant during the simulation period.

The more recent Lucas critique (1976) might also apply, because the models lack microfoundations in the neoclassical sense. The basic idea of the Lucas critique is that macroeconomic parameters derived from econometric studies describing aggregate behaviour of the agents may change if the economic policy changes, if these parameters are not "structural" that is grounded in the rules governing the behaviour of individual agents. The parts of the Lucas critique related to rational foresight of the agents have not been accepted by Post Keynesians and some New Keynesians (as pointed out by Goutsmedt, Pinzón-Fuchs, Renault & Sergi, 2017). Lucas assumed that Friedman's permanent income hypothesis correctly describes the aggregate consumption function and that firms operate in an environment where long-term investment is not subjected to fundamental uncertainty about the rate of return on capital. Post Keynesians also reject the interpretation of Philips curve and the explanation of causes of inflation provided by Lucas (an alternative theory of inflation was mentioned in Section 2.6).

The core of the Lucas critique in the general sense may however be valid as when the economic policy changes, individuals may consciously change their behaviour to avoid the consequences of the policy. The growth of the shadow economy in Greece after the introduction of the austerity packages is one of the most striking examples of these processes. Medina and Schneider (2018) have estimated that the shadow economy grew from 23.2% of the GDP in 2008 to 28.39% in 2012. It can be argued that a model based on neoclassical microfoundations would not capture these changes as human behaviour is far more complex than just intertemporal maximisation of utility by a single representative agent. Macroeconomics is dealing with the phenomena which cannot be correctly anticipated by analysing the behaviour of a single individual, as they emerge when multiple agents interact together. None of the neoclassical economic models has correctly anticipated the depth and persistence of recession induced in Greece by the introduction of austerity packages.

The following are possible limitations of the SFC methodology:
If there are missing state variables, missing causal links or oversimplified functional relationships, adjusting other parameters may lead to a model which seems to behave as desired under certain circumstances but leads to an incorrect understanding of the macroeconomic reality. This point applies to all economic models in general and explains why Keynes was reluctant to express his theories in a closed mathematical form, despite being a brilliant mathematician.

- The behavioural parameters which are assumed to be constant in the model may change in time (Keynes, 1939).

- Human actors may render economic policy changes ineffective by undertaking actions in anticipation of these changes (Lucas, 1976). Responses of the system can still be simulated assuming a range of possible changes in the underlying human behaviour but this may not be good enough for long-term forecasting.
- Due to the presence of stochastic disturbances (risk) and unexpected events (uncertainty), the forecasting accuracy of the models diminishes quickly over time. It is possible to introduce stochastic elements to an SFC model or perform a parametric sweep but the majority of Post-Keynesian economists, who distinguish between "risk" and "fundamental uncertainty" (Lavoie, 2014, p.73), do not use this methodology. Neoclassical and New Keynesian dynamic stochastic models do not generate more accurate forecasts than deterministic SFC models but Post-Keynesian economists are more aware of the limitations of their models.
- Dynamic SFC models, like all macroeconomic models, are unable to endogenously simulate the phenomena emerging from complex interactions between individual agents associated with speculative bubbles and shifts in investor or consumer confidence. These phenomena can be simulated by the making the relevant variables exogenous.

While more or less successful attempts to debunk Lucas critique have been made, it is hard not to accept the arguments raised by Keynes. The fact that the underlying structural models of the economy were incorrect explains why so-called Keynesian dynamic models of the 1970s turned out to be highly inaccurate, not only in regards to forecasting the accelerating inflation in the context of distributional conflicts, exchange rate wobbles and accelerating capital flows of the mid-1970s. Retrofitting "microfoundations" to these models to address the issues raised by Lucas (1976) fixed nothing (which became evident after the crisis of 2008).

Keynes did not build any complete mathematical models of his "General Theory" (1936), despite being a highly-skilled mathematician. He was critical about early attempts to formalise his verbal description of the macroeconomic processes presented in his "General Theory" in a form of a complete mathematical model such as the

Hicksian IS/LM. This model was actually rather an attempt to reconcile his theory of effective demand with the views of the neoclassicals and ignored the Z,D analysis of point of effective demand, provided in Chapter 3 of "General Theory" (Keynes, 1936) .

Parameters of a well designed dynamic macroeconomic model can be adjusted so that it reproduces the trajectory of a macroeconomic variable within the period used to train the model. This does not guarantee that the trajectory outside of that period will resemble the actual historic trajectory as multiple parameters can change in an unexpected way, the accuracy of estimation of the parameters may be inadequate and missing elements of the model may not allow for capturing the relevant phenomena. Theoretical models should simply not be used for forecasting.

There is nothing in the Keynesian critique of Tinbergen's methodology (1939) that would invalidate the idea of building a complete theoretical model, allowing for the validation of the macroeconomic theories and observation of the emergent phenomena. Both critiques (Keynes, 1939; Lucas, 1976) impose restrictions on how the results of the simulations can be interpreted. It is obvious that a theoretical SFC model is not a forecasting or a fiscal policy calibration tool. Someone may want to build a proper econometric model based on the experience gained by working with a theoretical SFC model.

The fact that actual trajectories of the evolution of macroeconomic variables can be reproduced with a reasonable accuracy by a model does not prove that the model is correct. It only shows that it might be correct or partially correct but it is up to the reader of this thesis to judge whether theoretical arguments about how the economy works are convincing enough or not. This does not mean that we should not attempt to build these models and use them in the same way as a biologist builds and uses a theoretical mathematical model of a dynamic biological system, to advance our understanding how the interaction of simple components leads to complex behaviour of the whole system.

Many of the SFC models contain a simplified one-good production system. While they do not rely on Cobb-Douglas or Constant Elasticity of Substitution (CES) aggregate production functions (used in DSGE models), the use of a single consumption and capital good is not a realistic approximation of the real economy.

The one-good production system presented by Godley and Lavoie (2007) correctly describes the distribution of revenue but it is too simplified to simulate more complex phenomena related to dynamic changes in the production processes such as reswitching, identified in the debates around the Cambridge capital controversy or changed in the capital intensity of production processes related to technological progress. The one-good production system may be however "good enough" to simulate the phenomena related to changes in the aggregate demand.

3.6 Comparing discrete-time and continuous-time models

3.6.1 The features of discrete-time models and the advantages of their use

The discrete-time macroeconomic modelling methodology evolved from early econometric models (Tinbergen, 1940). Keynes (1939) provided a critique covering, among other things, the statistical methodology used in determining the causality of the model. Some of his concerns may have been addressed only recently by the use of Dynamic Principle Component Analysis (Andrle et al., 2017). Early theoretical dynamic models of the business cycle were also expressed in discrete time (Kalecki, 1954). For a first-order difference equation system, the inter-dependencies between variables from the current and next steps can be depicted in the following way (see also Tinbergen, 1940, p. 74, Chart 1).

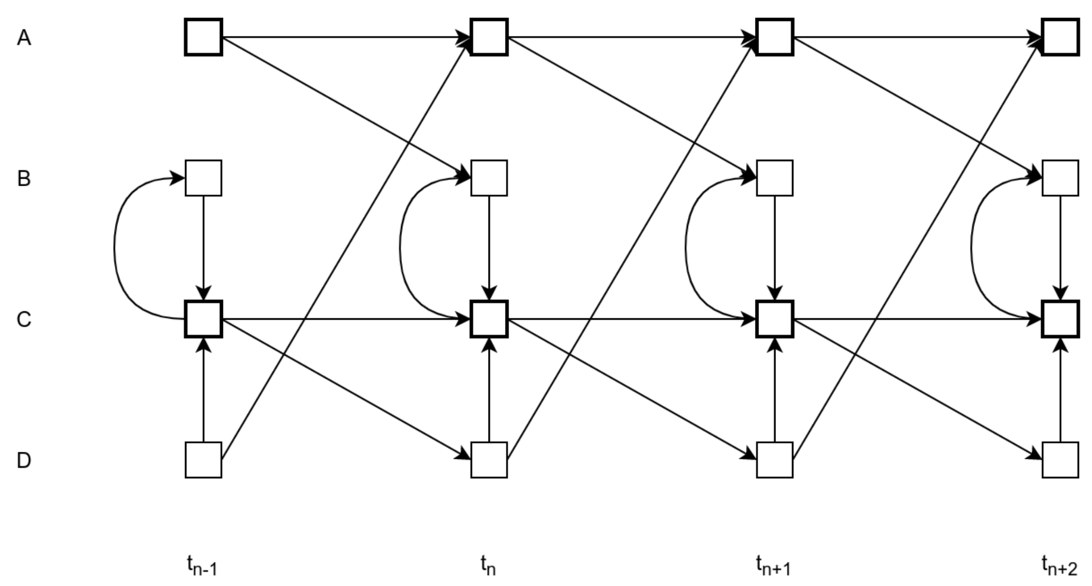

Figure 14: Relationship between variables A..D evolving over time in a discrete-time macroeconomic model

Obviously values from step “n” cannot depend on values from step “k” where $k > n$ (time step “k” is later than “n”) but circular dependencies can exist within the current time step because of the way parameters have been specified. This corresponds to short-term equilibrium defined in Section 2.2 which may cover some cases of circular dependency identified by Keynes (1939). Only these variables from step “n” which directly influence variables from latter steps are “state variables”. On the diagram above these would be “A” and “C”. The state of the model at $t=t_n$ (time “t” at step “n”) is fully determined by the values of A_n and C_n. The model might be defined as a system of 4 (possibly) non-linear algebraic-difference equations with values of some variables from step “n” and state variables from step “n-1” appearing as arguments to the functions.

The classical Marxian model of the production cycle (M-C-P-C'-M') is a discrete-time model. A discrete-time framework makes period analysis possible (as “current period” and “previous period” are defined) what allows for clear determination of the causality. Keynes and Kalecki could have described the saving – investment equality as an “ex-post” not “ex-ante” identity because they could demonstrate which process (for example investment) is driven by autonomous decisions of economic agents and which process (for example saving) is an accounting artefact. It is not true that saving and investment are equalised "ex-ante" by the market of loanable funds (that savings are re-lent, see Section 2.3). But there is no "ex-post" and "ex-ante" in continuous time. It is therefore difficult to anchor the spending of economic agents today in the context of the volume of income acquired yesterday.

The continuous-time models created by Steve Keen have been described by Thomas Palley (2014) as Post Keynesian-Monetarist. This is because aggregate demand has to depend on money velocity (Keen, 2015) as (in the absence of delayed processes often used by Michał Kalecki) this is the only available tool linking a particular flow "now" with anything else – in this case "anything else" must be a stock. But Keynes argued precisely against this kind of thinking as only some fraction of the total stock of money is in "circulation" (satisfying the transaction demand for money, see Section 2.3) while the rest remains dormant in saving accounts or stored as cash (Keynes, 1936, Chapter 15). Portfolio allocation decisions in the Keynesian liquidity preference theory change in time and there is no such a thing as a constant "money velocity" determining a rigid casual dependency of the volume of current spending on the size of the stock of money. During the recent Global Financial Crisis quantitative easing policy affected long-term interest rates and put a floor under prices of certain financial assets but it did not cause a significant change in aggregate demand and failed to produce a welcome increase in the inflation rate.

3.6.2 The disadvantages of the discrete-time approach

Despite a lot of effort, Stock Flow Consistent (SFC) models using discrete-time framework have never exceeded the level of sophistication which was achieved early in their history, when W. Godley and M. Lavoie published "Monetary Economics" (2007). This might be attributed to the opacity of large systems of algebraic-difference equations used to define these models. In order to build a new model by reusing an existing one implemented by someone else, the original model has to be thoroughly understood. Reading and reverse-engineering discrete-time models in order to understand the modelled processes is difficult.

- Stock and flow variables can be confused in discrete-time models because the time step is implicit.
- The identification of state variables is much more difficult in discrete-time models.
- In these models gradual adjustment processes (such as the adjustment of adaptive expectations expressed as a first-order ordinary differential equation) may be confused with processes which are delayed in time (such as the delivery of new units of productive capital, driven by investment decisions made several months earlier).

If at least one time delayed process is present, the model is described by a system of delay differential equations (DDE) instead of ordinary differential equations (ODE). Michał Kalecki (1954, p. 54) used this abstraction for modelling the consequences of investment decisions, for example current profits "P" would depend on past investment "I", with a delay "ω".

$$P_t = f(I_{t-\omega}) \quad (1)$$

Specifying initial conditions for DDEs and analysing the behaviour of solutions is far more difficult than for ODEs. A time delayed process can be approximated with multiple lagged processes (described by ODEs).

3.6.3 The features of continuous-time models and the advantages of their use

The following arguments for using the continuous-time framework in macroeconomic models have been provided in the first chapter of Gandolfo (1993)

- It is natural to treat a stream of individual economic decisions as a continuous-time stochastic process due to the great number of events and their non-synchronised nature
- It is easier to express dynamic adjustment processes in continuous time than in discrete time

- The estimator of models is independent of the observation interval (the time constant of the first-order linear system can be expressed in absolute time units rather than relative observation intervals)
- In discrete models there is no obvious time-interval that can serve as a "natural" unit – artefacts of the observation interval may appear in the models. It is necessary to test the invariance of results with respect to the period length (which can tend to zero in continuous-time framework)
- Adjustment functions may have a very high adjustment speed with respect to the observation period. It is impossible to estimate the adjustment speed based on the data.
- It is easier to model distributed-lag processes
- Differential equation systems are easier to handle than difference equation systems from the analytical point of view
- The simulations for any time interval are available in continuous-time framework.

The continuous-time framework is widely used in describing and simulating physical, chemical and biological systems, electronic circuits, control systems and technological processes (Fritzson, 2014). The most well-known analogue circuit simulation system, SPICE, uses a continuous-time framework. Modelica language has been widely accepted as an industry standard for simulating dynamic processes. Original System Dynamics models can also be simulated using a library implemented in Modelica. OpenModelica is an open-source implementation of Modelica standard (Fritzson, 2014). Modelica evolved from Dymola. Dymola was created as a language and simulation environment for large dynamic systems (Elmqvist, 1978). Modelling and simulation of dynamic systems plays a significant role in modern engineering. There is no compelling reason not to apply the same modelling methodology and the same tools to macroeconomic systems.

The continuous-time framework has been chosen to define and implement the models in this thesis. The OpenModelica modelling and simulation environment has been determined as the most suitable tool to run the simulations.

3.7 The use of difference and differential equations in dynamic SFC modelling

This section deals with some of the most difficult issues in modelling. While its content would be easy to understand for someone who is familiar with Control Theory or System Dynamics, it might be hard to read for an economist and may make little sense to a "pure" mathematician, for whom it will appear as a collection of disjointed statements. The author's goal is to link the modelling methodology commonly used in Control Theory and "hard" empirical sciences with the principles of Stock Flow Consistent modelling, initially developed in the discrete-time framework. It is not about developing new mathematical tools but about finding the most suitable existing mathematical language to define dynamic models of the macroeconomic reality. There is no need to treat macroeconomics in any special way and it can benefit greatly from the methodology used by engineers and scientists. Commonly used simulation tools are more than good enough to support building and running Stock Flow Consistent models.

3.7.1 The properties of models defined using implicit and explicit systems of differential equations.

Viejo Garcia, Gonzalez de Durana, Barambones, and Kremers (2011) provide a distinction between two approaches in modelling complex electric-mechanical systems, paradigms called Dynamic Systems and System Dynamics. System Dynamics was developed by Jay W. Forrester in the 1950s.

In Control Theory systems are often presented as not changing in time (time-invariant) but reacting to external signals. In Post-Keynesian economics dynamic systems often have no external inputs but their parameters change over time. The formulae presented below refer to non-autonomous systems (model parameters may change in time), without external inputs.

System Dynamics models use systems of explicit ordinary differential equations (ODEs) to define a model.

$$\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t));\boldsymbol{x}(0)=\boldsymbol{x_0} \tag{2}$$

where $\mathbf{x}$(t) is the vector of state variables $[x_0,x_1,..,x_n]^T$ and $\mathbf{x}_0$ is the vector of initial values.

The Dynamic Systems approach is based on using systems of implicit differential algebraic equations to describe a dynamic process.

$$\boldsymbol{f}(t,\boldsymbol{x}(t),\frac{d\boldsymbol{x}(t)}{dt},\boldsymbol{y}(t))=\mathbf{0};\boldsymbol{x}(0)=\boldsymbol{x_0} \tag{3}$$

where $\mathbf{y}$(t) is a vector of variables which are not integrated (are not state variables).

System Dynamics models can be considered to be a subset of Dynamic Systems models. It is not possible to convert all Dynamic Systems models described by systems of implicit differential algebraic equations into System Dynamics models described by systems of explicit ordinary differential equations.

3.7.2 The features of traditional discrete-time SFC models

The majority of SFC models, such as model GROWTH from Godley and Lavoie (2007), have been specified as systems of difference-algebraic equations. In general implicit systems of difference-algebraic equations are described by the following formula:

$$\boldsymbol{F}(t,\boldsymbol{X_{t\text{-}1}},\boldsymbol{X_t})=\mathbf{0} \tag{4}$$

where $\mathbf{X}_t$ is an n × 1 vector of variables X at a time index “t” and $\mathbf{X}_0$, the vector of initial values, is provided.

SFC models are often defined in the discrete-time domain by providing equations determining values of all variables (also see Figure 14).

$$\boldsymbol{X_t}=\boldsymbol{F}(t,\boldsymbol{X_{t\text{-}1}},\boldsymbol{X_t}) \tag{5}$$

Some of the elements of the vector of variables **X** at a time "t" may depend not only on the values of elements at a time "t-1" (the previous sample) but also on the values of other elements at a time "t" (the current sample). This corresponds to a dynamic short-run equilibrium evaluated at a time "t".

Generally there is no obvious distinction between flows and stocks in discrete-time models. Econometric software packages such as Modler and Eviews not only allow for graphing the data and calculating regressions but also support the running of discrete-time simulations. The modelling methodology used in Eviews has been described in detail in Brillet (2011). Programs for econometric modelling use standard numerical algorithms such as Newton's (or Broyden's) method to solve the system of non-linear equations (evaluate the dynamic equilibrium) at each time step. Then the values from the step "t" are propagated to step "t+1" and the procedure is repeated.

A dynamic system described in the discrete-time framework by a system of algebraic-difference equations (4) can usually be described in the continuous-time framework by a system of algebraic-differential equations (3).

3.7.3 The link between dynamic SFC models and Control Theory

In Control Theory, a dynamic system can be described using a state space representation.

> *A system can be considered as an abstract operator mapping from the input signal space to the output signal space.*
>
> *A description that uses the state signal is called a state-space description or state-space model. State-space models for lumped (concentrated-parameter) systems consist of two sets of equations:*
>
> *1. State equations, which describe the evolution of the states as a function of the input and state variables, being a set of time-dependent ordinary differential equations.*

2. Output equations, which relate the value of the output signals to the state and the input signals, being algebraic equations. (Hangos, Bokor & Szederkényi, 2004, p. 18, 24).

A dynamic non-autonomous (time-varying) system with external inputs can be described by the following system of equations:

$$\begin{aligned} &\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t),\boldsymbol{u}(t)) \\ &\boldsymbol{y}(t)=\boldsymbol{h}(t,\boldsymbol{x}(t),\boldsymbol{u}(t)) \\ &\boldsymbol{x}(0)=\boldsymbol{x_0} \end{aligned} \tag{6}$$

where **u**(t) is the input vector, **x**(t) is the state vector, **y**(t) is the output vector and $\mathbf{x}_0$ is the vector of initial values (as illustrated in Figure 15 below).

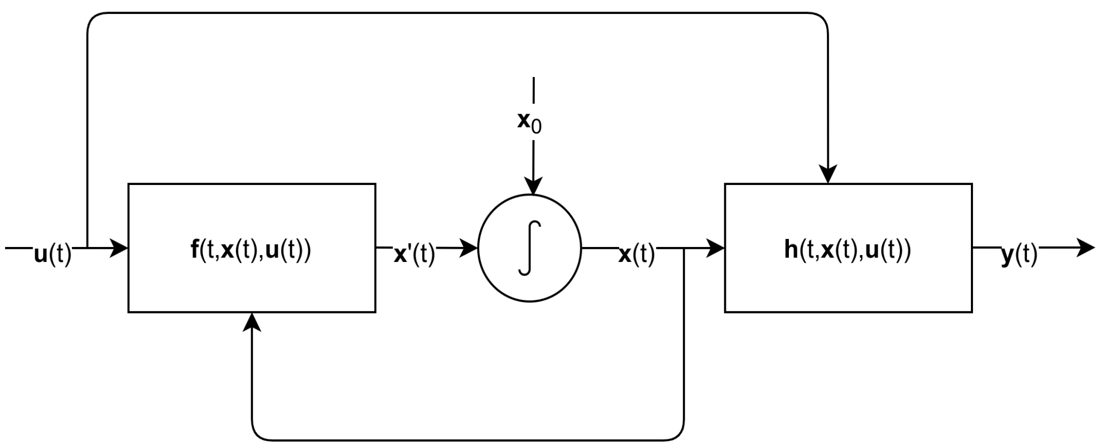

Figure 15: A dynamic non-autonomous system with external input.

A dynamic SFC model defined without introducing markets in a short-run equilibrium is usually defined without external inputs and is described by the system of equations (7), as illustrated in Figure 16.

$$\begin{aligned} &\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t)) \\ &\boldsymbol{y}(t)=\boldsymbol{h}(t,\boldsymbol{x}(t)) \\ &\boldsymbol{x}(0)=\boldsymbol{x_0} \end{aligned} \tag{7}$$

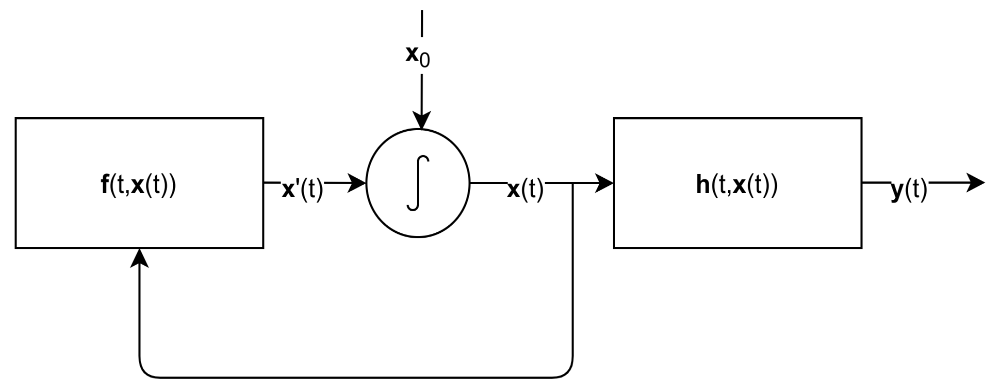

Figure 16: A dynamic non-autonomous system without external input.

This description corresponds to System Dynamics abstraction provided in (2), extended with the mapping of the state vector to output signals.

Components (building blocks) of a dynamic multisectoral macroeconomic model such as the firms sector or the households sector are dynamic systems with inputs, as represented by **u**(t) in system of equations (6), connected to outputs of other components (Hangos et al., 2004, p. 32, modified). This is illustrated in Figure 17 below.

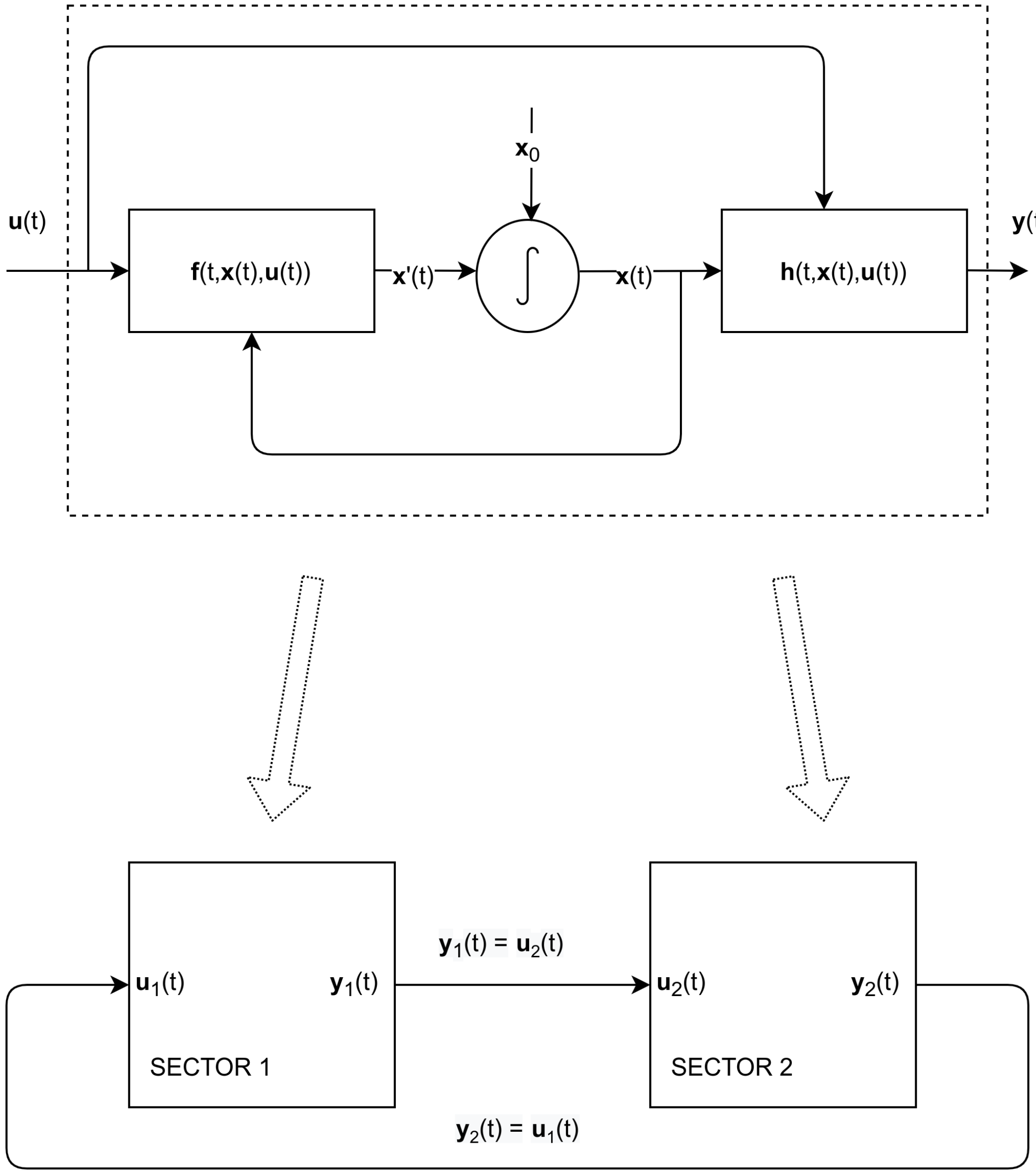

Figure 17: A multisectoral model presented as a dynamic non-autonomous system.

If a (multisectoral) SFC model is built from several building blocks is often easier to define it using a system of equations (8). This is because an output signal produced by one sector may be an input signal of another sector (as illustrated in Figure 18 below).

$$\begin{aligned}&\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t),\boldsymbol{y}(t))\\&\boldsymbol{y}(t)=\boldsymbol{h}(t,\boldsymbol{x}(t))\\&\boldsymbol{x}(0)=\boldsymbol{x_0}\end{aligned} \tag{8}$$

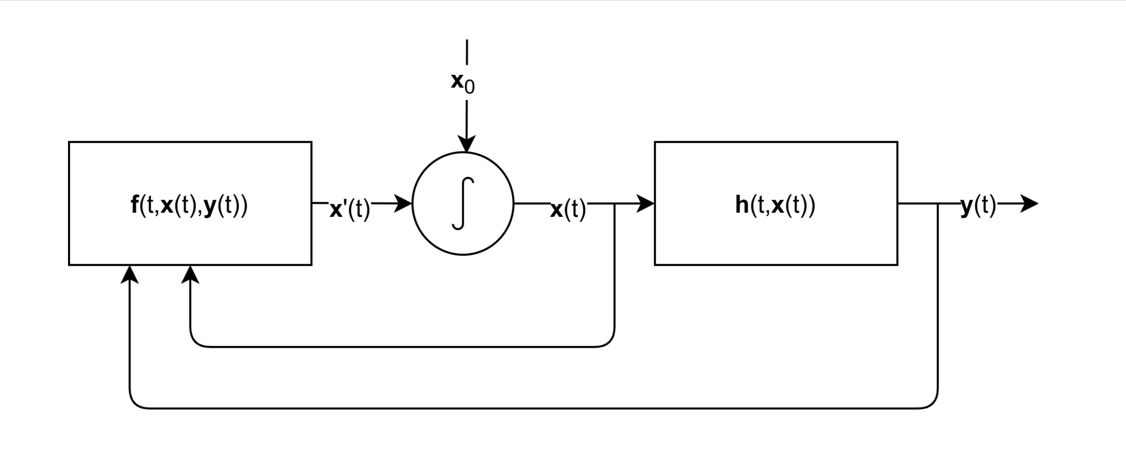

Figure 18: A dynamic non-autonomous system described by system of equations (8) .

3.7.4 SFC models with markets in short-run equilibrium

The process of market-clearing (in Post Keynesian economics this usually applies only to financial markets) is that they are described by a system of "simultaneous equations" which brings together several prices, stocks and flows. An example of such a system, a "portfolio decisions" block (the Tobin asset demand system), is provided (in a discrete-time form) by Godley and Lavoie (2007, p.395). The model is then described by system of equations (9) instead of (8). The signal flow graph is presented in Figure 19 below.

$$\begin{aligned}&\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t),\boldsymbol{y}(t))\\&\boldsymbol{y}(t)=\boldsymbol{h}(t,\boldsymbol{x}(t),\boldsymbol{y}(t))\\&\boldsymbol{x}(0)=\boldsymbol{x_0}\end{aligned} \tag{9}$$

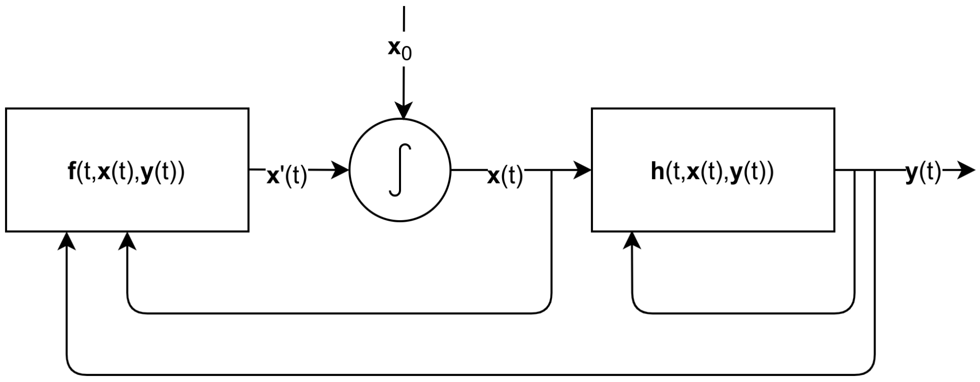

Figure 19: A dynamic non-autonomous system described by system of equations (9).

The system of equations (9) is a special case of a generic system of implicit algebraic-differential equations (3). Usually the system used to define a macroeconomic SFC model is not canonical and the vector **y**(t) contains not only output variables but also some "auxiliary" variables which have been introduced to improve the readability of the model. The model is usually created by writing down equations defining the values or the rates of change of all individual variables used in the model. All $dx_i(t)/dt$ and $y_i(t)$ (where "i" are indices of the scalar variables) have to appear on the left hand side of the individual (scalar) equations, later aggregated to a vector form seen in formula (9).

In some cases it might be possible to convert the system of equations (9) to a form described by formula (7) by analytically solving the second vector equation in (9) for **y**(t) and then substituting the value for **y**(t) in the first equation, which would lead to an explicit system of ODEs (Jorissen, Wetter & Helsen, 2015). This may however be impractical or impossible. One of the key limitations of simulation tools used in System Dynamics is the lack of the ability to solve implicit systems of algebraic-differential equations.

It may be possible to avoid using simultaneous equations in the model of the financial markets by adding expectational variables (see Section 3.7.5) however this would over-complicate the model. In actual financial markets, lags in trading individual commodities are very short (microseconds or even hundreds of nanoseconds to milliseconds) due to the presence of high frequency traders. Obviously at the high level of aggregation this does not matter but a model with artificial lags would be even more difficult to understand.

The system of equations (9), defining an SFC model, can always be converted to the following form:

$$\begin{aligned}
&\frac{d\boldsymbol{x}(t)}{dt}=\boldsymbol{f}(t,\boldsymbol{x}(t),\boldsymbol{y}(t))\\
&\boldsymbol{g}(t,\boldsymbol{x}(t),\boldsymbol{y}(t))=\mathbf{0}\\
&\boldsymbol{x}(0)=\boldsymbol{x_0}
\end{aligned}\tag{10}$$

The system of equations (10) defines a Hessenberg index-1 implicit (semi-explicit index-1) system of differential-algebraic equations (Campbell, Linh & Petzold, 2008). The implicit function theorem requires the function **g**() to be continuously differentiable for **y** and the Jacobian matrix of **g**() for **y** to be invertible (non-singular) in order for the relation **g**()=0 to be convertible into a function in a region of a space bounded by a circle. This means that special care is needed in defining functions describing short-term equilibrium in terms of smoothness.

A system of equations describing a continuous-time model has to be integrated in discrete time. Numerical methods used for the integration are described in the literature mentioned by Campbell et al. (2008). These algorithms have been implemented in multiple software tools, used in engineering and science. The modelling and simulation environment used in this research is OpenModelica (Fritzson, 2014).

Only a limited subset of capabilities offered by OpenModelica will be utilised in simulating dynamic SFC models. It is possible to build a graphical diagram depicting connections between components of a physical system and allow the modelling program to do the rest – create a system of differential-algebraic equations and transform them to the form suitable for iterative evaluation and integration. Since the creation of a suitable graphical representation of components of a macroeconomic model would take too much effort, models will be directly defined as systems of algebraic and differential equations in the language called Modelica.

3.7.5 State variables in SFC models

The intuitive meaning of "state variables", defined as a vector **x**(t) in equations (6), (7), (9) and (10) is as follows: let us imagine that we lose all the information about the model of an economic system. We need to know which variables contain all the information required to restore the full knowledge of the system at a certain point of time. These variables are state variables and their number is equal to the number of degrees of freedom of the system (Terman & Izhikevich, 2008). Obviously there might be constraints on the values of variables so that the system is stable or its state makes sense in macroeconomic sense (for example unit wage must be positive). A macroeconomic model may have more variables and equations than state variables (it may not be canonical). In the case of non-canonical models it may be possible to choose which variables are state variables.

If an individual market has a unique equilibrium, it has one degree of freedom. If a central bank sets the interest rate on an interbank market, it has to supply the quantity of bank reserves determined by the demand from the banking sector; it cannot control both the interest rate and the quantity of reserves.

State variables in discrete-time SFC models are those elements of the vector X which appear inside the function F(.) in equation (4) with time index "t-1". A variable whose previous value determines the current value of the same or another variable is a state variable. Usually these variables are stocks but anything (also a flow) can be a state variable in a discrete-time model. For example in model SIM (Godley & Lavoie, 2007, p.81) current consumption (at a time "t") depends on disposable income in the previous period (at a time "t-1"), which in turn depends on consumption in the same period (at a time "t-1").

A state variable in a continuous-time model is defined with the use of a derivative of the variable on left hand side of (2) or with the use of a derivative of the variable inside **f**() in (3). These equations define the rates of change of state variables in time. What is integrated to calculate the value of a stock (starting from an initial value) is a flow. If real stocks are measured in [kg], real flows can be measured in [kg/s]. If nominal stocks are measured in [USD], nominal flows can be measured in [USD/year]. This highlights the problem with using flows as state variables. Unlike in discrete-time models, a "previous" value is not known as the time step is infinitesimally small. What replaces the previous value is the rate of change.

In a simple model without lags, if current disposable income is included among the state variables of the model, current consumption depends on current disposable income but current disposable income also depends on current consumption and the value is undefined due to a circular dependency (see Section 3.6.1). This problem can be easily resolved by adding expectational variables. The processes of updating adaptive expectations can be described by a first-order ordinary differential equation (Gandolfo, 1993). Expectational variables store information about the current state of the system available to the agents. A consumer forms expectation about how much disposable income is available now (based on how much has been recently available; this is how adaptive expectations are built). This information is required to decide how much to spend "now". Current spending determines the current disposable income (assuming no other lags in a simple model). It is also possible to introduce time delays to store the "previous value" of a flow (see Section 3.6.2), but this leads to greater difficulty in running the simulation.

State variables in a SFC model should therefore be either stocks or expectational variables. An alternative approach, consistent with the System Dynamics approach, has been presented in Andresen (1998) but is strictly monetarist (expectational variables do not exist whereas monetary stocks are used as state variables and spending depends on the quantity of money possessed by agents). This approach is rejected by Post Keynesians, as pointed out by Lavoie (2014, Section 8.1).

Chapter 4 Modelling the GFC and its Aftermath

4.1 Theoretical foundations of the model

4.1.1 The role of theoretical models in explaining economic phenomena

According to Bailer-Jones (2009) the scientific process (building theories describing natural phenomena) involves building a hierarchy of models, with each playing a different role in the process.

It can be argued that what we consider as phenomena, manifesting themselves in the patterns of data, depend on the theories used to describe the reality. *"Not only is it not always clear how to delineate a phenomenon, shedding doubt on the claim that phenomena are natural kinds, but it is also sometimes hardly possible to interpret data without having a phenomenon in mind."* (Bailer-Jones, 2009, p. 167)

The process of creating theories describing reality involves harvesting data by performing experiments or, if this is impossible or impractical, by making observations of the examined processes. But patterns in the data, which can be identified using statistical methods, do not explain themselves. Models are used to link the data and the phenomena which are suspected of generating the observed patterns. It is hard to deny that phenomena exist independently of being investigated but they can only be identified when causal factors explaining their existence or emergence are proposed. This is the role of the models. "*In order to capture a phenomenon, the phenomenon is modeled, and the way the phenomenon is modeled will influence how the phenomenon is defined. Theory provides the background for the model development.*" (Bailer-Jones, 2009, p. 170)

The following categories of models have been identified: data models, models of the experiments and theoretical models. The function of data models is converting raw data into a canonical form, enabling its use for the validation of theoretical models. Statistical (econometric) models belong to this category. A model of the experiment enables experimental testing of a hypothesis presented in a theoretical model. The role of theoretical models is capturing the phenomenon by providing a description and highlighting the factors which are considered to be relevant for constituting the phenomenon.

> *Raw data cannot serve to confirm a theoretical model about a phenomenon, but have to undergo procedures of data analysis and be put into the form of a data model to be usable for an empirical test. Thus empirical confirmation takes place between data model and theoretical model, not between data and phenomenon, and also not between data and theoretical model.* (Bailer-Jones, 2009, p. 173)

Theory can only be applied to the phenomenon using a theoretical model which can be used to support and confirm the theory by linking it to the data model. There needs to be a strong link between the empirical evidence and the theoretical model but a theoretical model may not be empirical as the indirect link with the actual raw data can be provided by a separate data model.

The goal of this thesis' study is to build a macroeconomic model of dynamic processes which have developed in the economy of the United States, in order to explain the phenomena, not to predict precise outcomes. The limitations of macroeconomic modelling from the epistemological point of view have been presented in Section 3.1. The model of American economy defined in Chapter 4 belongs to the category of theoretical models. The design and calibration of the model can be influenced by the methodology used in System Dynamics or "Cyber-Physical" modelling (Fritzson, 2014), applied to empirical sciences and engineering.

4.1.2 The short-run equilibrium in the model and the long-run trajectory

The modelling approach used in this research is based on building a model of equilibrium growth of an idealised economy and then simulating various scenarios by changing the parameters and applying exogenous shocks.

A real economy is never in a state of equilibrium growth or "long-run equilibrium" (see Section 2.2) as stock-flow norms such as the household wealth to GDP ratio keep changing. The economy is only in a state of short-run equilibrium characterised by aggregate supply matching aggregate demand at certain prices. It is assumed that quantities would mostly provide adjustment on goods and labour markets while prices would adjust on financial markets.

If a stable model is left undisturbed it will converge towards an equilibrium growth trajectory. The stability often depends on the choice of initial values of state variables. Models which are not asymptotically stable may generate a stable endogenous cycle or may be unstable. The model running the baseline scenario simulation is assumed to be stable.

The model is supposed to explain the widening of the demand gap caused by the changes in income distribution (see Figure 12). According to Bhaduri and Marglin (1990) and Cynamon and Fazzari (2015) the redistribution of income from low-income households to high-income households has resulted in lower marginal propensity to spend (where spending includes consumption and induced investment) and as a consequence, the widening of the demand gap (lowering the GDP), as already mentioned in Section 2.7. This effect is illustrated as a transition from Y1 to Y2 on Figure 20, depicting a Keynesian cross, which is a simplified graphical representation of a short-run equilibrium.

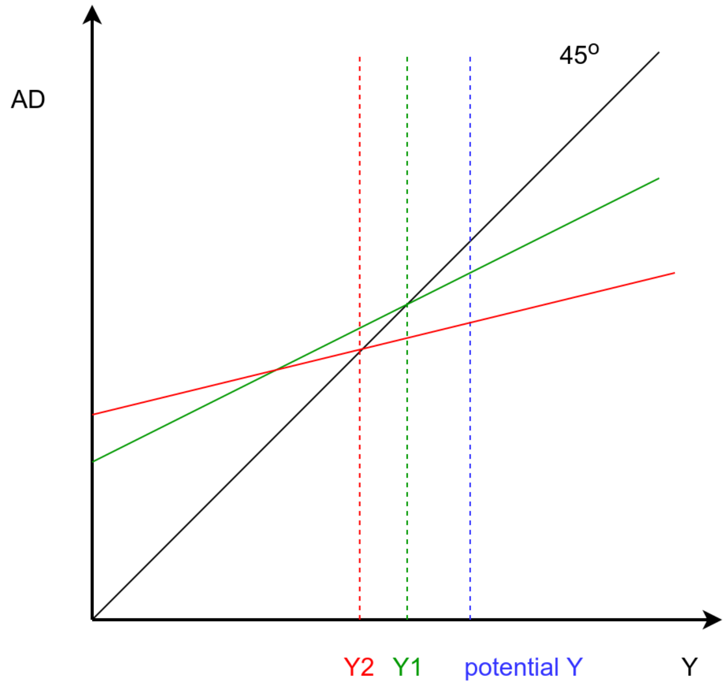

Figure 20: Short-run changes in the aggregate demand caused by changes in income distribution.

AD is aggregate demand, Y is aggregate income.

This demand gap was temporarily filled by the autonomous expenditure. The first stage of this process involved the dot-com bubble (see Figure 3 and Figure 4). The second stage involved the housing bubble (see Figure 2 and Figure 3). It can be argued that the dot-com bubble involved over-investment in productive capital. The housing bubble clearly involved the process identified by Fiebiger and Lavoie (2017) as a temporary increase in "non-capacity generating semi-autonomous household expenditures" (see Section 2.8).

The theoretical concept of the Sraffian Supermultiplier has been introduced in Section 2.8. The supermultiplier effect combines together the multiplier effect related to marginal propensity to consume and the accelerator effect related to induced investment. The short-run operation of the supermultiplier is depicted in Figure 21. The increase in autonomous expenditure shifts the aggregate demand line upwards (the orange line). The induced investment amplifies this effect (the purple line). The intersection of the purple line (the schedule of aggregate demand as a function of aggregate income) with the 45-degree line determines the value of aggregate income in a short-run equilibrium. The GDP increases from Y2 to Y3, partially closing the demand gap.

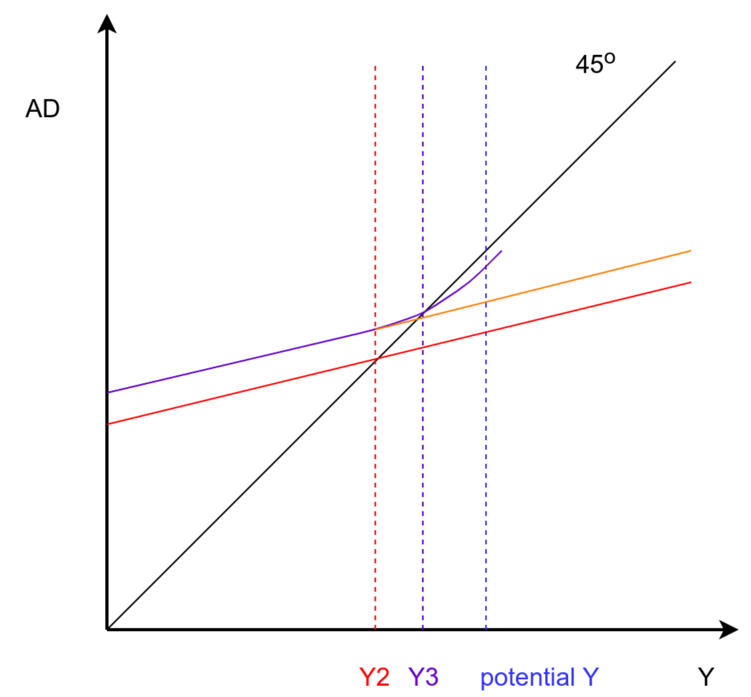

Figure 21: Short-run changes in aggregate demand caused by autonomous household or government expenditure.

AD is aggregate demand, Y is aggregate income.

The long-run growth trajectory is obtained by modelling the accumulation processes (as discussed in Section 2.2). A period of lower employment and lower productivity growth results in a downward shift of the potential GDP growth line (see Figure 1), because temporary losses become incorporated into the trend growth line – as observed by Summers (2014). In 2018 the American economy had finally resumed its trend growth but it is unlikely that it will ever recover to its pre-crisis growth asymptote.

A Modigliani consumption function (defined in Godley & Lavoie, 2007 p.75), is used in the model. In the long run, the aggregate demand is co-determined not only by the level of disposable income but also by the wealth of the consumers (in the short run the expected wealth does not change much, except for financial wealth, as it is mainly determined by slow capital accumulation processes). The pace of growth of stock of firms capital and real labour productivity coincides with the pace of growth of the aggregate demand in this demand-driven model (as shown in Table 13).

Consumers develop a taste for new products, which are invented due to technological progress and introduced to the market, so there is no saturation of consumer needs with durable goods, as people want more products when a greater variety is available.

4.1.3 Overview of the model

The model of the GFC and Secular Stagnation is based on the heavily modified model GROWTH from Godley and Lavoie (2007), which has been chosen as the starting point in creating the continuous-time model because it is sophisticated enough to simulate the relevant macroeconomic phenomena. It is a long-run growth model with capital accumulation determined by a dynamic short-term equilibrium. As discussed in Section 3.6.3 the continuous-time approach makes the understanding of the model easier and allows for capturing of the system dynamics at a time scale shorter than one year (the period often used in discrete-time models).

The goal of the research is to build a theoretical model resembling the reality in order to investigate the links between various macroeconomic processes and phenomena, not to build a forecasting model used to predict the future and calibrate possible policy responses. Instead of trying to directly fit the model parameters to existing econometric data, a more indirect approach, inspired by Godley and Lavoie (2007), has been chosen.

The following approach to building the model, described by Fritzson (2014, p.569) as "bottom-up modelling" has been used:

> *First we formulate the basic equations and design small experimental models on the most important phenomena to gain a basic understanding of the application area. After some experimentation we have gained some*

understanding of the application area, we can then add more details and restructure our model fragments into a set of of low-level model components which can be used as building blocks for further modeling. These component models might have to be restructured several times if they turn out to have problems when used for applications. We gradually build more complex application models based on our component models, and finally arrive at the desired application model.

Using the graphical (diagrammatic) modelling approach, fully supported in the simulation environment (OpenModelica), turned out to be too time consuming due to the need to define custom graphical components and has thus been abandoned. The model has been defined as a collection of algebraic and differential equations. However the iterative integration, calibration and validation approach, borrowed from the modern simulation and product design methodology, described by Fritzson (2014) in Section 1.7, has been fully adopted. The theoretical and practical issues related to building and calibrating the model are discussed in Sections 4.1.1 and 4.1.5.

First, a model of stable exponential growth has been built and calibrated, using historic data from a relatively undisturbed period in the mid-1980s. This scenario, using fixed values of all the parameters, is the baseline. Equilibrium growth is characterised by constant values of stock-flow norms and exponential changes of other variables while the rates of growth of these variables remain constant. Values of these stock-flow norms are determined by the values of the parameters.

In the second stage of modelling, several constant parameters of the baseline model have been made exogenous variables and changed during the simulation period, in order to simulate various scenarios. This approach allows for running experiments demonstrating the impact of changes in parameters on the behaviour of the whole economy. Several well-known macroeconomic phenomena have been reproduced.

Finally, simulation of the actual trajectories of GDP and unemployment has been attempted. Section 4.1.4 contains more details of how the collection of simulation scenarios has been developed.

The model GROWTH presented in Godley and Lavoie (2007) has been subjected to the following set of modifications:

- The household sector has been dis-aggregated into low-income and high-income households. Each class has a separate consumption function schedule. This allows for the simulation of distributional changes and is closer to the reality than a model effectively based on a single representative household.
- High-income households derive a significant portion of their income not just from capital (in the form of profits and capital gains) but also from labour. This is an element of novelty in the model as the traditional framework used for analysing income distributional issues divides the society into workers and capitalists. As shown by Atkinson, Piketty and Saez (2011) and by Elsby, Hobijn and Şahin (2013), rising remunerations of managers and professionals (labour income) have played a significant role in the growth of the share of income of the top-earners.
- Residential real estate has been added as a separate category of assets, its unit price consists of the variable price of a block of land and the price of a building, linked with the price of the single good produced in the model economy. This allows for simulation of the housing bubble and the following slump, by allowing for prices of land to fall when demand for new housing drops.
- Net Acquisition of Financial Assets has been separated from Net Acquisition of Financial Liabilities as in Barwell and Burrows (2011): some agents borrow while others save in a bank or repay their loans. The original model uses "deflated net lending" (Godley & Lavoie, 2007, p. 393). This modification allows for correct accounting for an increase in aggregate demand caused by debt-financed household expenditure. In a period of time all the money borrowed is spent while the amount of money repaid is subtracted from disposable income which only partially reduces aggregate demand. This is due

to (effective) marginal spending propensity of the aggregated households sector lower than one.

- The wage adjustment function is asymmetric, as depicted in Figure 23. The asymmetry is related to wage stickiness when the unemployment rate is high, mentioned by Krugman (2018) in the context of the GFC. Keynes (1936), critiquing the views of Pigou on unemployment, observed that money-wages are rigid and do not fall in a recession, preventing the labour market from clearing. However, when the unemployment rate is low, workers can demand pay rises.
- Unemployment benefits have been added as transfer payments. This is required to correctly account for the reduction in aggregate demand caused by an increase in unemployment.
- The population and workforce are growing. This allows for separating the results of productivity growth from the results of population growth.
- The growth in productivity is endogenous and slows down when unemployment is high. This allows for getting more realistic trajectories of unemployment and GDP after the GFC. According to Summers (2014), Secular Stagnation was also caused by a decrease in the rates of growth of labour productivity.
- A simplified foreign sector has been added. This is required to correctly simulate the GDP trajectory.
- Some simplifications of the interactions between the banking sector, households and the government have been implemented. To look at the major changes in aggregate demand it is not essential to look at the allocation of assets between short-term and long-term government debt.

The following diagram depicts the structure of the model.

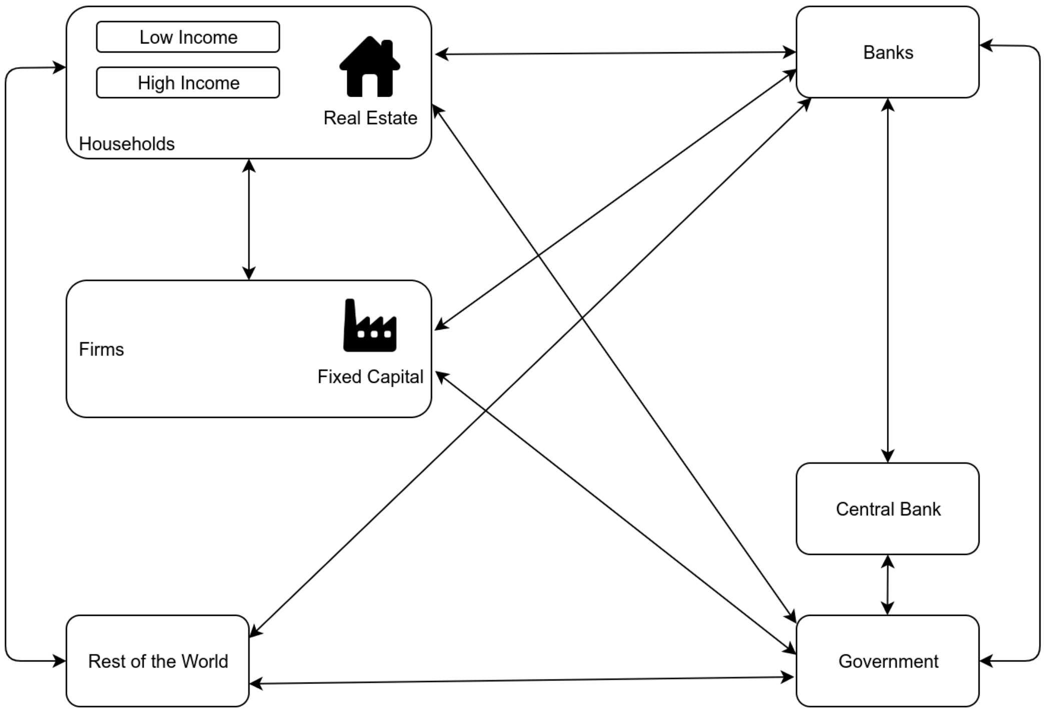

Figure 22: The macroeconomic sectors, real assets and interactions between the sectors in the model.

4.1.4 Building the model and designing the simulation scenarios

The central idea behind building the stable growth model first and running scenario simulations later is the separation of the relatively stable subsystem of production, consumption and productive investment from an exogenous source of instability mainly in the form of debt-financed residential investment and acquisition of durable goods (as argued by Mason (2018). This corresponds to the idea of explaining the business cycle by fluctuations of non-capacity generating semi-autonomous expenditures interacting with the supermultiplier, presented by Fiebiger and Lavoie (2017).

The model based on the idea of the supermultiplier effectively isolates the sources of instability and the chaotic economic cycle from the components which would demonstrate asymptotically stable behaviour. A baseline scenario can be modified to investigate the impact of exogenous shocks (such as changes in government spending or in mortgage-financed real-estate investment undertaken by households) and the impact of changes in internal parameters such as the distributional parameters. Fiebiger and Lavoie (2017) argue that all the business cycles in the last 50 years (except for the dot-com bubble) can be mainly attributed to changes in autonomous household spending.

Long-term deviations from the stable growth trajectory can be attributed to changes in the internal parameters of the production-consumption subsystem such as the falling share of the income flowing towards low income households and fiscal withdrawal (while the external sector is draining aggregate demand due to current account deficits). Changes in real labour productivity would also affect the long-term growth trajectory.

Some of the scenarios simulated with the model correspond to experiments demonstrating the influence of changes in exogenous parameters on dynamic trajectories generated by the model. The final goal of the simulation is reproducing the actual historic macroeconomic phenomena known as the Global Financial Crisis and Secular Stagnation.

The model is not designed to simulate the underlying long-run processes, belonging to the category of "political economy" phenomena, affecting the evolution of the finance sector and the real estate market. It is clear that changes in the regulatory framework (the relaxation of lending criteria) allowed for debt-financing of the speculative activities on the real estate market. These changes occurred within a wider context of the financialisation of the global economy. Debt securitisation played a significant role in hiding the systemic risk posed by subprime loans, as mentioned by Bernanke (2010).

Unlike in the period before the 1990s, the supply of land became inelastic when demand increased in the early 2000s. There had been periods of increased building activity in the US before but they did not lead to a housing bubble. In the model it is assumed that before the GFC the total value of real estate assets was linked with the stock of mortgages, as shown in Figure 6. This is supported by data since the early 1990s. Further details of this assumption are provided in Section 4.2.

It is assumed that the parameters describing the consumption function do not change in time and changes in the volume of consumption are only caused by changes in disposable income and the expected stock of wealth. Human actors can adjust their expectations but their behavioural patterns do not change significantly during the period of time which is simulated by the model. This is a very strong and crude assumption and in fact behavioural changes affecting the consumption function within income groups do occur, as documented by Summers, Carroll and Blinder (1987). Since the model of the economy has more parameters than state variables, changes in the GDP can also be explained by endogenous changes in the behaviour of consumers and investors. This hypothesis is not supported by the econometric studies mentioned by Fiebiger and Lavoie (2017). It has been clearly identified that the economic system responds to exogenous stimuli in a way which does not significantly change over time. It is impossible to learn with reasonable confidence from existing econometric studies how much "deep" and "hidden" behavioural parameters change over time. Even if these changes contribute to the changes in some of the macroeconomic variables, they were not as dominant as the direct changes in aggregate demand as demonstrated by Andrle et al. (2017).

The model has been specifically designed to simulate the following phenomena:

- The "secular stagnation" that is the decline in the rate of growth of the real GDP after the GFC (Figure 1). This could be linked to distributional changes (Figure 12) and to the reduction of the share of government consumption and investment expenditures in the total demand for goods and services (Figure 10). The reduction in the ratio of productivity growth after the GFC (Figure 5) has also contributed to the stagnation. These changes were documented, among others by Elsby et al. (2013). Cynamon and Fazzari (2015) suggest that this had caused a decline in the household demand which was masked for a certain period of time by debt-financed spending. The distributional changes could have been caused by the decline in the share of low-income wages in the total volume of wages, as shown in Figure 4 in Elsby et al. (2013), and the flattening of the effective taxation scale.

- The stock marked (dot-com) bubble (Figure 4), which temporarily increased the household wealth to GDP ratio (Figure 7). The bubble caused a temporary increase in gross corporate investment (Figure 2).
- The housing bubble and the GFC (Figure 1). These phenomena could be explained by looking at the changes in net lending (mainly financing construction activities) on the level of economic activity and employment (Figure 2 and Figure 3). The actual financial crisis was caused by the loss of bank equity caused by the insolvency of borrowers related to the drop in the price of real estate assets. The model is not designed to capture the details of the financial crisis at this level of granularity but rather to show the effects of changes in autonomous household expenditure, government expenditure and changes in wealth induced by falling prices of real estate (Figure 6 and Figure 7), flowing through the supermultiplier.
- The fiscal government intervention (Figure 10) and the changes in monetary policy (Figure 8), reducing the depth of the downturn.

The simulations have been divided into several groups. The baseline scenario has been calibrated and tested in Sections 4.6 and 4.7. The model has been dynamically calibrated to achieve a realistic value of a short-run spending multiplier in Section 4.9. Simple long-run trajectories of economic growth have been simulated in Section 4.10. Simulations involving changes in single parameters have been described in Section 4.11. More complex scenarios (a stock market bubble and a real estate bubble) are included in Section 4.12. A collection of scenarios reflecting the actual historical changes has been introduced in Section 4.13. Possible long-run recovery scenarios have been included in Section 4.14. Finally, selected dynamic phenomena have been simulated in Section 4.15.

The following dynamic simulation scenarios have been implemented:

- **Reference** – the baseline (stable) scenario, with balanced growth (used in all simulation groups).

- **FiscalStimulus** – simulates short-run effects of a temporary increase of the government expenditure to GDP ratio, allows for the estimation of the spending multiplier, used in Sections 4.9 and 4.15.
- **FiscalExperiment** – simulates long-run effects of a reduction of aggregate demand caused by a permanent reduction of the government expenditure to GDP ratio, used in Sections 4.10 and 4.11.1.
- **ProductivityExperiment** – simulates long-run effects of a reduction in the rate of productivity growth, used in Section 4.10.
- **TradeBalanceChanges** – simulates short-run effects of changes in the trade balance, allows for the estimation of the foreign trade multiplier, used in Section 4.11.4.
- **MonetaryStimulus** – simulates short and long-run effects of a change in the rate of interest, allows for the investigation of the transmission channels of the monetary policy in the model, used in Section 4.11.3.
- **DistributionalChanges** – simulates long-run effects of distributional changes, used in Section 4.11.2.
- **StockmarketBubble** – simulates short-run effects of changes in the portfolio choice parameters and normal capacity utilisation, used in Section 4.12.1.
- **HousingBubble** – simulates short-run effects of changes in the demand in housing markets
- **HousingBubblePriceCrash** – simulates short-run effects of changes in the demand in housing markets and changes in land prices.
- **HistoricalNoBubblesNoStimuli** – simulates a combination of distributional changes and limited fiscal withdrawal, used in Section 4.13.
- **HistoricalNoStimuli** – simulates a combination of distributional changes, limited fiscal withdrawal, stock and housing market bubbles.
- **HistoricalNoFiscalStimulus** – simulates a combination of distributional changes, limited fiscal withdrawal, stock and housing market bubbles and a monetary stimulus.
- **HistoricalGFC** – simulates a combination of distributional changes, stock and housing market bubbles, a monetary stimulus and historical fiscal policy. The

real GDP trajectory closely resembles the actual historical data. Used in Sections 4.13 and 4.15.

- **FiscalExpansion** – built on top of "HistoricalGFC", simulates an attempt to stimulate the economy affected by Secular Stagnation by increasing the government expenditures to GDP ratio, used in Section 4.14.
- **IncomeRedistribution** – built on top of "HistoricalGFC", simulates an attempt to stimulate the economy affected by Secular Stagnation by redistributing income towards low-income households by adjusting effective low-income and high-income tax rates, used in Section 4.14.

All the historical scenarios have the trajectory of labour participation driven endogenously as a detailed simulation of the labour market falls outside of the scope of this research.

The list of variables shocked in the experiments is presented in Table 15.

4.1.5 Calibration of the model

The dynamic SFC model built to simulate the GFC and Secular Stagnation belongs to the category of theoretical models, as explained in Section 4.1.1. It is not a data model. Econometric SFC models such as Burgess et al. (2016) exist but their goal is not the same as the current study.

It would be tempting to use realistic values of key behavioural parameters such as the consumption function schedule taken from literature to define the model. Unfortunately, this approach simply does not work. Multiple studies produced different values of consumption propensities for various income groups. Econometric models based on different assumptions produce different values. The majority of neoclassical and New Keynesian data models use the underlying theoretical model of a representative rational agent maximising its utility to produce a function which parameters are then estimated using regression from the actual raw data. Even the values of the spending multiplier, estimated from the actual data in DSGE models, often fall below one. This has been highlighted by Blanchard and Leigh (2014) when the failure of "expansionary austerity" policy in Europe has become apparent. The alternative approach is to assume (or adjust) the values of the parameters so that the model simulates the "stylised facts". This approach has been adopted in the current study.

One may question the calibration procedure involving reconstructing the coefficients used in the consumption function so that it produces a realistic value of the spending multiplier by stating that we may be assuming our results. But as long as the values produced by this calibration method are not inconsistent with what has been produced by other studies and the simulations (experiments with the model) reproduce other phenomena, not used for calibration, this calibration procedure is still methodologically sound in the context of building a theoretical model.

Even if the actual values of state variables and parameters are known, using them to initialise the model directly does not work as the nonlinear solver used in OpenModelica fails to produce a solution of the system of implicit nonlinear equations, describing the initial state of the model. The problem is with the model itself as it describes an idealised and simplified economy, not the real one, but this does not invalidate the model. The goal is to find the values which work and which still make macroeconomic sense. Bailer-Jones (2009, p.174) clearly distinguishes between the modelled phenomena and the model:

> *What is taken to be the phenomenon becomes somewhat reconstructed in the course of the modeling process. The modeled phenomenon may depart somewhat from what the phenomenon started out to be taken to be when it first attracted curiosity and was first studied. Despite this perhaps uncomfortable air of constructivism, however, it is the investigation of the phenomenon that results in data about the phenomenon, and the data subsequently serve as a constraint for the model. The link of the model to empirical evidence is required to remain strong. While there is an empirical link, how we delineate and describe a phenomenon is invariably also linked to the way we have learned to model it.*

With the level of complexity of the model required to reproduce the relevant phenomena it is impossible to use nonlinear regression to determine the values of all the parameters. There are more parameters than state variables in the model. The assumption of equilibrium growth in the baseline scenario allows for reasonably easy calibration based on macroeconomic data. It is assumed that the core part of the economy is stable. This part is calibrated first. Then exogenous shocks are applied to the modelled economy. This approach is consistent with the Sraffian Supermultiplier approach, isolating the parts which are stable from the parts which create instability, as mentioned in Section 4.1.4.

Reasonable values of model parameters taken from the Post Keynesian literature, especially from Godley and Lavoie (2007), have been used in the baseline scenario, as described in Section 4.6. Some of these parameters were modified during the model development and calibration procedure, using the iterative approach mentioned in Section 4.1.3.

An obvious limitation of using historic data in calibrating the model in the baseline scenario is that some debt-related stock-flow norms measured during the period of time used for calibration are inconsistent with the values of other parameters, because the economy was not in a state of equilibrium growth. Some “stylised facts” about an “idealised economy” have to be assumed. It has been assumed that the American economy in the second half of 1980s was close to the equilibrium growth path, but this was obviously not the case.

Knowing the equilibrium growth path, it may be possible to solve the system of equations for the stock-flow norms by specifying additional conditions on derivatives of the stock-flow norms. Even with the moderate number of equations present in the current model this would be a time-consuming task, especially if undertaken simultaneously with model debugging and modifications. The values of stock-flow norms consistent with the equilibrium growth have not been observed because the economy was never in that state, so this calibration method is not viable.

It is much easier to let the model run with realistic initial values and harvest the values of state variables after a few hundred years, when the model settles on its stable growth trajectory. These values of state variables can then be normalised (scaled) so that the population and nominal GDP are consistent with the "stylised" actual data. Another parameter which can be chosen arbitrarily is initial labour productivity, which affects the price level and links, together with the nominal GDP, the corresponding real and nominal variables. The number of equities can also be chosen arbitrarily. The ratios of the remaining state variables in the state of equilibrium growth are determined by the parameters of the model. The growth ratios of these variables are shown in Table 13.

If the values of other variables resemble the actual data from the calibration period (knowing that we are referring to an "idealised" economy, assumed to be in dynamic equilibrium, not the real American economy in 1985), the model parameters relevant to the equilibrium grow path have been estimated correctly. While this procedure is too crude to build a model suitable for short-term forecasting, it is nevertheless fit for purpose, if the model is only used to demonstrate the causality of the phenomena rather than calculate their magnitude, which is the goal of building this theoretical model.

The model also needs to show correct dynamic behaviour in regards to changes in the government expenditure and the interest rate set by the central bank. The procedure of modifying the values of relevant parameters and re-normalising the model (to make its equilibrium growth path consistent with the assumed "stylised facts") has been repeated until the dynamic behaviour of the model has also become consistent with the results of econometric studies chosen to calibrate the model.

4.1.6 Choosing the simulation period

The simulation of the processes occurring in the American economy over the last 20 years requires calibrating the baseline model with relatively recent data.

In the mid-1980s the American economy had relatively low ratios of the stock of public and private debt to GDP inherited from previous periods of time, yet other parameters related to income distribution, trade balance and the relative size of government spending, had just started changing.

The simulation starts in 1984 (after the end of the 1981 recession, caused by the Volcker's shock linked with disinflation) and finishes in 2019. Year 1984 has been chosen because the American economy had already stabilised then after high inflation in the 1970s and the subsequent deep recession used to break a wage-price spiral. Yet the income distribution changes linked to the emergence of monetarism and supply-side economics only started appearing in 1984 so stock-flow norms still resemble the values from the era of Keynesian macroeconomic management (1945-1973).

It is therefore assumed that the values from 1984 (or averages from that period of time) correspond to an idealised economy which was exponentially growing in a state of near-equilibrium. This assumption may not be entirely realistic but it is nevertheless possible to find a set of parameters and initial values of state variables with magnitudes consistent with the real data.

4.2 Detailed model assumptions

The model is based on model GROWTH from Godley and Lavoie (2007) however significant changes described in Section 4.1.3 have been made.

- The baseline model simulates an economy resembling the United States in the period 1984-2019 (see Section 4.1.6).
- Relative changes of the parameters of the same magnitude as in the statistical data will be used in simulating various scenarios even if the assumed starting values of the parameters are not exactly the same as the measured values.
- Households have been arbitrarily divided into low-income and high-income as in the SFC growth model with a housing market developed by Zezza (2008).
- In the model, the total target demand for labour in the economy is calculated first, based on real GDP and average real productivity - (37) and (38). The actual total demand for labour adjusts to target demand with a variable lag (39). It is assumed that the net increase in employment takes more time than net job losses

when the companies go bankrupt or have to reduce the workforce in a recession. This phenomenon is analysed in detail by Bivens and Shierholz (2010), the main reason in lagged hiring is related to accelerated changes in productivity when the economy resumes growth. For simplicity these elements have not been included in the model. The assumption about different lags is also required in the model to generate an unemployment rate trajectory resembling in shape the historical one depicted in Figure 2.

- A fixed fraction of demand is satisfied by high-income households, supplying "complex labour" (41). The rest is satisfied by low-income households, supplying "simple labour" (40). The labour demand is proportionally divided between high and low income households. The demand for labour corresponds to the wage bill. Wages (salaries) allocated might not reflect the allocation of the quantity of labour measured in time units per period of time. Since no attempt is made to reduce complex labour to simple labour, it is sufficient to provide a formula for allocation of labour income as a fraction of total demand for labour for both income groups. This fraction together with the rate of distributed profits is one of the key income distribution parameters, slowly changing in time.
- The labour force participation rate is driven exogenously (34) (see Figure 13), as the labour market model is too simplified to allow for simulation of all of the relevant phenomena such as the movements into and out of the labour force.
- If the total demand for labour expressed in monetary terms (38) is lower than the labour supplied, unemployment is allocated proportionally to both income groups so that there is no change in income distribution - (31) and (32).
- The real wage rate aspirations function (generating the Philips curve) is asymmetric and has a horizontal segment as depicted in Figure 23; (29) and (30). Downward nominal wage rigidity has been explained in Section 4.1.3. In the model, in the absence of a significant reduction of real wages and a fall of the markup rate, prices and nominal wages remain unchanged.

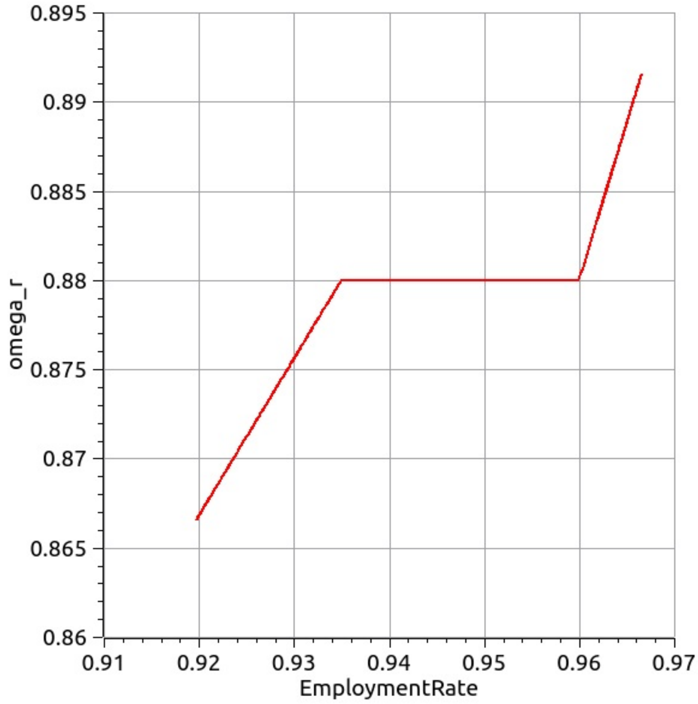

Figure 23: Real wage aspirations rate ω_r as a function of the rate of employment ER.

- High-income households derive a significant portion of their income (60) not just from capital (in the form of profits and capital gains) but also from labour (41). The demand for labour supplied by managers and professionals cannot be reduced to the demand for simple labour. There is an effective separation of labour markets.
- A Modigliani-like aggregate consumption function defined on p. 75 in Godley and Lavoie (2007) has been assumed – see (85), (89), (90) and (91). Since the Sraffian Supermultiplier model is demand-driven, this function together with the induced investment aggregate function (32), (17) and (20), effectively determines the responses of the non-government sector to changes in autonomous components of aggregate demand.
- The aggregate consumption function of all households is a sum of consumption functions of high-income and low-income households (91). The absence of the "middle class" in the model simplifies it without losing the ability to demonstrate the relevant phenomena but also makes the calibration of the coefficients more difficult. The aggregate consumption function is related to the "life-cycle income" hypothesis but it does not share its micro-foundations.

- High-income households consume a fixed fraction of their expected stock of wealth in a unit of time - (84) and (85). Their consumption does not depend directly on their current disposable income. This might be consistent with the "permanent income" hypothesis approach however the model assumes adaptive rather than rational expectations (in a stock-flow consistent modelling environment agents can only build their expectations using information available at the current time and cannot predict the future). One may consider a more realistic assumption that marginal propensity to consume out of current disposable income is not 100% for the low-income class and 0% for the high-income class. Unfortunately, in a simplified two social class model, allowing for a more realistic assumption yields an incorrect value of the fiscal spending multiplier. We would need to split the households into more income classes if we wanted to implement a more sophisticated consumption function which is based on econometric data.
- It is assumed that high income households know the growth trend of the economy which allows them to correctly estimate the undisturbed wealth growth trajectory (84). In regards to unexpected (unanticipated) changes they have adaptive expectations with a slow pace of adjustment if the wealth growth accelerates and with a faster pace of adjustment if the wealth starts falling.
- Due to the difficulty in finding reliable econometric data and the choice of the simulation period, the behavioural coefficients of the consumption functions defining the behaviour of both income groups have been assumed to be constant. Changes in income distribution are the only source of the changes of the parameters of the aggregate consumption function. However introducing yet another layer of complexity would not make the model more realistic if we do not know the exact values of the coefficients of the consumption function.
- It is assumed that low-income households consume (almost) all of what they have earned (90). Low-income households do not receive any capital income but may be receiving unemployment benefits (68). They only maintain a cash buffer determined by the level of their consumption (explained by the Keynesian "transaction motive") - (74), (77) and (90). Low-income households buy

consumption goods or invest in real estate assets, borrowing money from the banking sector (101).

- High-income households have more complex saving and investment functions. The Tobin-like portfolio allocation function - (96), (97) and (98), describing the behaviour of high-income households does not include equities in liquid assets. Only these assets (currency, bank money and bills) which can be freely moved between different classes in the short run are included. Investing in real estate is also financed by mortgage borrowing (100).
- A fraction of the total consumption demand is satisfied by imports (136). The model describes a semi-closed economy but the actual trade imbalances of the US over the last 50 years were significant, as shown in Figure 9 (trade deficit exceeded 6% of the GDP in 2006). It has been assumed that the average import propensity is a fraction of the average consumption propensity. That fraction is in the model a slowly changing (exogenous) control variable. The coefficient has been calibrated so that the resulting trade balance resembles the actual data, as shown in Figure 82. Modelling the processes affecting the trade balance such as the changes in the exchange rates and the prices of imported commodities (oil) would require introducing an open economy model.
- Prices are "cost-determined" in the model (45). For simplicity, it is assumed that the single commodity produced by the virtual economy is a "finished good" and companies do not reduce their markup when the demand falls, as explained in Section 2.2. While this assumption may seem unrealistic in general, it could be applied to a highly-developed and relatively closed American economy after the end of oil crises of the 1970s. The mechanism of determination of prices is the same as in the GROWTH model, presented by Godley and Lavoie (2007).
- The quantity of shares is inelastic and determined by supply (the need of firms to finance new investment) - (56).
- For simplicity capital losses or gains on government securities are excluded from the model.
- Treasury bills are used to approximate interest-yielding government debt securities in general. There are no long term government debt securities (bonds) in the model.

- For simplicity, the Central Bank only holds the amount of debt securities which is required to match the demand for the currency in the form of cash and bank reserves - (135).
- Banks buy up all the remaining government debt securities - (143).
- Banks only have to fulfil their capital adequacy requirement (147) and (152), as enough liquidity is always provided by the Central Bank.
- For simplicity it is assumed that the interest rate on bank deposits is equal to the interest rate on government debt securities. Banks set their own lending interest rate based on the target profit rate - (151) and (146).
- It is assumed that the rate of return on a bank's capital is the same as the rate of return on fixed capital of firms (defined as the ratio of distributed profits to the value of the company's shares) - (149). This departs from the assumption made in Godley and Lavoie (2007, p. 401), that dividends of banks are a fixed fraction of the GDP.
- Total household borrowing (99) is determined to be a fraction of total disposable income. It is split between low income households and high income households. For simplicity only mortgage lending is simulated.
- There are no loan defaults.
- The growth of the population (workforce) is simulated - (33). The microeconomics of the housing market is not analysed in the baseline model. Both population growth and the changing size of households are factors creating housing demand. In the baseline model the demand for housing is exogenous.
- The value of real estate assets consists of the price of land and the price of goods embedded in residential building structures - (126). In the model (during the growth phase) the rate of growth of house prices is the same as the rate of growth of the stock of mortgage loans - (120), (121) and (122). The ratio of the price of a housing structure to the price of the consumer good remains constant, while the price of land is variable (see Figure 6). For simplicity the housing demand, the mortgage lending and repayment volumes are exogenous.
- In the baseline model both income groups do not change their relative debt positions (high-income households borrow a fixed fraction of the total volume of mortgages and they repay them at the same pace) - (100), (101), (102) and (103).

- Both high and low income households have the same construction spending behaviour, spending a fixed fraction of what has been borrowed - (114) and (115). The relationship is empirical. From the microeconomic point of view newly originated loans are spent on construction of new structures and on re-sales of existing dwellings. The loans repayment is a separate process. A fraction of spending on dwellings goes to existing landowners and does not involve paying for labour and profits of the construction industry.
- Mortgage refinancing is ignored.
- Time constant parameters taken from the discrete-time mode GROWTH in Godley and Lavoie (2007), have been adjusted to more realistic values.

4.3 Symbols used in the model

Symbols are based on (Godley and Lavoie, 2007, p. ix-xvii), superscripts have been replaced by subscripts (for example instead of s^e, s_e is used). Lower-case variables are real, upper case are nominal. Symbols are ordered as in an OMEdit Plotting window.

4.3.1 Exogenous parameters

ER_{max}	High employment rate threshold for wage expectations growth
ER_{min}	Low employment rate threshold
ER_{pr1}	High employment rate threshold for productivity growth
ER_{pr0}	Low employment rate threshold for productivity growth
GYR	Government expenditures to GDP target ratio
HILD	Fraction of labour demand satisfied by high-income households
HILE	Fraction of lending going to high-income households
HIN	Number of high income individuals to total population ratio
IHMOV	Construction spending to mortgage origination volume ratio
LF_N	Labour force to total population ratio
NCAR	Normal capital adequacy ratio of banks
Ω_3	Nominal wage adjustment parameter
REMOR	Real estate value to mortgage debt stock ratio

Θ_f	Profit tax rate
Θ_{hh}	Personal income tax rate, high income households
Θ_{hl}	Personal income tax rate, low income households
UBR	Unemployment benefits to wages ratio
α_2	Propensity to consume out of wealth;
β	Expected real sales adjustment coefficient
δ	Rate of depreciation of fixed capital
δ_{rep}	Household loans repayment rate
δ_{RES}	Rate of depreciation of residential buildings
ε	Expected real regular disposable income adjustment coefficient
ε_{GYR}	Government expenditure adjustment coefficient
ε_M	Actual markup adjustment coefficient
ε_{vh}	Expected real wealth of high-income households adjustment coefficient during growth periods
ε_{vhr}	Expected real wealth of high-income households adjustment coefficient in a recession
η	New loans coefficient
η_{LD}	Employment adjustment coefficient during growth periods
η_{LDr}	Employment adjustment coefficient in a recession
γ	Stock of inventories adjustment coefficient
γ_u	Real capital growth to capacity utilisation coefficient
gr_N	Population growth rate
gr_g	Growth rate of real government expenditures
gr_{pr0}	Baseline labour productivity growth rate
λ_{20}	Fraction of investible wealth of high-income households V_{fmah} allocated to B_h
λ_{22}	r_b elasticity of B_h/V_{fmah}
λ_{24}	r_K elasticity of B_h/V_{fmah}
λ_{25}	YD_{rh}/V_h elasticity of B_h/V_{fmah}
λ_{40}	Fraction of investible wealth of high-income households V_{fmah} allocated to equities p_e*e
λ_{42}	r_b elasticity of p_e*e/V_{fmah}

λ_{44} r_K elasticity of p_e*e/V_{fmah}

λ_{45} YD_{rh}/V_h elasticity of p_e*e/V_{fmah}

λ_{50} Fraction of foreign sector wealth allocated to B_{FS}

λ_b Bank dividends to GDP ratio

λ_c Cash to consumption ratio

μ Average net import propensity

ω_0 Independent wage expectations coefficient

ω_1 Coefficient determining reduction of wage expectations in low employment range

ω_2 Coefficient determining increase of wage expectations in high unemployment range

ψ_D Dividends to firm profits ratio

ψ_N Target financing of investment by new equity

ψ_U Target retained earnings to investment ratio

r_m Deposit interest rate set by Central Bank equal to bills interest rate

ρ Compulsory reserve ratio on bank deposits

σ_A Lending and deposit rates spread adjustment coefficient

σ_N Normal historic unit cost adjustment coefficient

σ_T Target ratio of inventories to sales

σ_{se} Normal historic unit cost adjustment coefficient

u_0 Normal stock of fixed capital / GDP ratio (normal capital utilisation)

4.3.2 State variables

B Government bills (representing all government securities)

LD Labour demand reduced to simple labour

L_f Loans to firms

L_{hh} Loans to high-income households

L_{hl} Loans to low-income households

N Total population

OF Own funds (bank capital)

V_{FS} Foreign sector nominal wealth

V_{Mh} High-income households' liquid wealth (excluding equities)

V_{Ml} Gross monetary assets (broad money) - low-income households

W Nominal wages

e Quantity of firm equities

g Real government expenditures

in Real inventory

k Real capital stock

φ Actual mark-up

pr Labour productivity

s_e Expected real sales

v_{he} Expected real wealth of high-income households

v_{RES} Real value of residential buildings

4.3.3 Other (non-state) variables

B_{FS} Bills held by foreign sector

B_b Bills held by banks

B_{cb} Bills held by the Central Bank

B_h Bills held by households

C Nominal consumption

CAR Actual capital adequacy ratio

C_h Nominal consumption of high-income households

C_l Nominal consumption of low-income households

CD Nominal consumption of domestically manufactured goods and services

ER Employment rate

FD_b Bank dividends

FD_f Firm dividends

FU_b Actual retained earnings of banks

FU_{bT} Target retained earnings of banks

FU_f Retained earnings of firms

FU_{fT} Planned retained earnings of firms

F_b Actual profits of banks

F_{bT}	Target profits of banks
F_f	Realised net profits of firms
F_{fT}	Net profits target of firms
G	Nominal government spending (excluding transfer payments)
GD	Nominal government debt
GL	Total gross household lending (mortgage origination volume)
GL_h	Gross household lending to high income households
GL_l	Gross household lending to low income households
H	High-powered money (currency)
H_b	Bank reserves
H_h	Total cash held by households
H_{hh}	Cash held by high-income households
H_{hl}	Cash held by low-income households
I	Total nominal gross investment
IM	Nominal net imports
IN	Nominal inventories
I_f	Nominal gross corporate investment
I_h	Total nominal gross housing investment (construction of dwellings)
I_{hh}	Nominal gross housing investment, high-income households
I_{hl}	Nominal gross housing investment, low-income households
K	Nominal capital stock
LD_T	Employment (labour demand) target
LS	Labour supply
L_h	Total loans to households (mortgages)
M	Total money deposits
M_{FS}	Money deposits of foreign sector
NES	New equity sales
NL	Total net household (mortgage) lending
NL_h	Net household (mortgage) lending, high-income households
NL_l	Net household (mortgage) lending, low-income households
NUC	Normal unit cost
OF_T	Own funds (bank capital) target

PE	Price to earnings ratio
PSBR	Nominal government deficit
REP	Total household (mortgage) loans repayment
REP_h	Household (mortgage) loans repayment, high-income households
REP_l	Household (mortgage) loans repayment, low-income households
S	Nominal sales
T	Total taxes
T_f	Corporate taxes on profits
T_h	Total personal income taxes
T_{hh}	Personal income taxes, high-income households
T_{hl}	Personal income taxes, low-income households
UB	Unemployment benefits
UC	Unit cost
V	Total nominal net wealth of households
V_M	Gross monetary assets (broad money) held by households
V_{MIT}	Gross monetary assets (broad money) held by low-income households, target value
V_{RE}	Total nominal value of real estate
V_{RES}	Total nominal value of residential buildings
V_{REh}	Nominal value of real estate owned by high-income households
V_{REl}	Nominal value of real estate owned by low-income households
V_{fmah}	Investible wealth belonging to high-income households
V_h	Nominal net wealth of high-income households
V_l	Nominal net wealth of low-income households
WB	Total nominal wage bill
WB_h	Nominal wage bill, high-income households
WB_l	Nominal wage bill, low-income households
Y	Nominal GDP, production approach
YD_r	Total nominal disposable income
YD_{rh}	Nominal disposable income, high-income households
YD_{rl}	Nominal disposable income, low-income households
YP	Total nominal household income

YP_h	Nominal household income, high-income households
YP_l	Nominal household income, low-income households
add_l	Lending mark-up over deposit rate target
c	Total real consumption
cd	Real consumption of domestically manufactured goods and services
c_h	Real consumption, high-income households
c_l	Real consumption, low-income households
gr_k	Real capital stock growth rate
gr_{pr}	Labour productivity growth rate
i	Real gross investment
i_f	Real gross corporate investment
i_h	Total real real estate investment (construction of dwellings)
i_{hh}	Real real estate investment, high-income households
i_{hl}	Real real estate investment, low-income households
im	Real net imports
in_T	Real inventory target
nl	Total real new lending (total mortgage origination) to households
nl_h	Real new lending (mortgage origination) to high-income households
nl_l	Real new lending (mortgage origination) to low-income households
ω_r	Real wage growth aspirations to productivity ratio
ω_t	Real wage aspirations
p	Normal cost pricing (price level)
p_e	Price of equities
φ_T	Actual mark-up target
π	Price inflation rate
q	Tobin's q ratio
r_K	Firms dividend yield (nominal rate of interest on capital)
r_l	Loan interest rate
rr_l	Real loan interest rate
s	Real sales
u	Capital utilisation
v	Total real net wealth of households

v_{RE}	Total real value of housing assets
v_{REh}	Real net value of housing assets owned by high-income households
v_{REl}	Real net value of housing assets owned by low-income households
v_h	Real net wealth high-income households
v_l	Real net wealth low-income households
y	Real output (GDP)
yd_r	Total real regular disposable income
yd_{rh}	Real disposable income, high-income households
yd_{rl}	Real disposable income, low-income households

4.4 Balance sheets of the economy sectors

The relationship between the sectors is shown in Figure 22.

Table 2: Balance sheet, low-income households

Assets	**Liabilities + Equity**
Re_l	L_{hl}
H_{hl}	V_l (equity)

Table 3: Balance sheet, high-income household

Assets	**Liabilities + Equity**
V_{REh}	L_{hh}
H_{hh}	V_h (equity)
B_{hh}	
M_h	
$e \cdot p_e$	
OF	

M_h is the stock of deposits owned by high income households, low income households do not have deposits (because of this, $M_h = M - M_{FS}$). Liquid assets consist of H_{hh}, B_{hh} and M_h. Households can choose between M_h and B_{hh} as the banking system can absorb the demand for bills – unless this demand is greater than the quantity of available bills.

Demand for equities affects their prices but shares cannot be sold (are illiquid). The value of OF (the stock of bank capital) also does not depend on households' portfolio allocation decisions.

Table 4: Balance sheet, firms

Assets	**Liabilities+ Equity**
K	L_f
IN	$e \cdot p_e$ (equity)

Table 5: Balance sheet, banks

Assets	**Liabilities + Equity**
H_b	M
B_b	OF (equity)
L	

Table 6: Balance sheet, central bank

Assets	**Liabilities**
B_{cb}	H

Table 7: Balance sheet, government

Assets	**Liabilities + Equity**
	B
	$-V_g$ (nominal equity)

Table 8: Balance sheet, foreign sector

Assets	**Liabilities + Equity**
M_{FS}	V_{FS} (nominal equity)
B_{FS}	

4.5 The model definition

All logarithms are natural: $\log(x) = \log_e(x)$.

All state variables are defined as derivatives in differential equations. The initial value of a state variable also needs to be specified in the model.

The equations based on the discrete-time model GROWTH presented in Godley and Lavoie (2007), are referenced in curly brackets "(G&L...)".

4.5.1 The production sector – aggregate demand, production and investment decisions

Real output *y* (G&L 11.1)

$$y(t) = s_e(t) + \gamma \cdot [\mathrm{in}_T(t) - \mathrm{in}(t)] \tag{11}$$

Expected real sales s_e (G&L11.2)

The firms determine expected real sales using a lagged adaptive expectations formula. The value of β has been increased from the original value 0.5 proposed by Godley and Lavoie (2007), so that adjustment time constant is about a month (β = 12). The original discrete-time formula contains a term including productivity growth but since β has been increased, reducing error in the steady state, the term with the productivity growth can be dropped.

$$\frac{d s_e(t)}{dt} = \beta \cdot [s(t) - s_e(t)] \tag{12}$$

Inventory target in_T (G&L 11.3)

The coefficient σ_T (a reciprocal of the time constant) determines how long the inventories would last with the expected level of sales if they were not topped up by production (the value $\sigma_T = 0.2$ corresponds to 2.4 months).

$$\mathrm{in}_T(t) = \sigma_T \cdot s_e(t) \tag{13}$$

Real inventory *in* (G&L11.5)

Real inventory is a state variable, its rate of change is equal to the real production minus real sales.

$$\frac{d\,\mathrm{in}(t)}{dt}=y(t)-s(t) \tag{14}$$

Real capital stock *k* (G&L11.6)

Real capital stock is a state variable, its relative rate of change (a logarithmic derivative) is determined by the rate of net investment.

$$\frac{dk(t)}{dt}=k(t)\cdot\log[1+gr_k(t)] \tag{15}$$

Rate of growth of real capital stock gr_k (G&L11.7, modified)

The rate of growth of real capital stock (rate of net investment) depends in the model on capital utilisation only (it does not depend on the interest rate), it drops to zero in a depression. If the rate of net investment is made sensitive to the rate of interest, the model becomes overly sensitive to the changes in monetary policy, as discussed in Section 4.11.3.

$$gr_k(t)=\begin{cases}(1+gr_{pr})\cdot(1+gr_N)-1+\gamma_u\cdot[u(t)-u_0], \\ \quad if\ (1+gr_{pr})\cdot(1+gr_N)+\gamma_u\cdot[u(t)-u_0]>1 \\ 0, if\ (1+gr_{pr})\cdot(1+gr_N)+\gamma_u\cdot[u(t)-u_0]\leq 1\end{cases} \tag{16}$$

Capital utilisation rate *u* (G&L 11.8)

$$u(t)=\frac{y(t)}{k(t)} \tag{17}$$

Real loan interest rate rr_l (G&L 11.9)

$$rr_l(t)=\frac{1+r_l(t)}{1+\pi(t)}-1 \tag{18}$$

Price inflation rate π (G&L 11.10)

$$\pi(t)=\frac{dp(t)}{dt}\cdot\frac{1}{p(t)} \tag{19}$$

Real gross corporate investment i_f (G&L 11.11)

The flow of real gross corporate investment does not include the changes in inventories, but the replacement of depreciated capital is included.

$$i_f(t)=[gr_k(t)+\delta]\cdot k(t) \tag{20}$$

Real gross investment *i*

The flow of real gross investment consists of the real gross corporate investment and the real households residential investment.

$$i(t)=i_f(t)+i_h(t) \tag{21}$$

Real sales *s* (G&L 11.12, modified)

The flow of real sales in the model includes only the consumption of domestic production, government expenditures and total real gross investment but excludes the consumption of imported goods.

$$s(t)=cd(t)+g(t)+i(t) \tag{22}$$

Nominal sales *S* (G&L 11.13)

$$S(t)=s(t)\cdot p(t) \tag{23}$$

Nominal inventories *IN* (G&L 11.14)

The value of nominal inventories is calculated at unit costs.

$$\mathrm{IN}(t)=\mathrm{in}(t)\cdot UC(t) \tag{24}$$

Nominal gross corporate investment I_f (G&L 11.15)

$$I_f(t)=i_f(t)\cdot p(t) \tag{25}$$

Nominal gross investment *I*

$$I(t)=I_f(t)+I_h(t) \tag{26}$$

Nominal capital stock *K* (G&L 11.16)

$$K(t)=k(t)\cdot p(t) \tag{27}$$

Nominal GDP *Y* (G&L 11.17)

The measure of Gross Domestic Product includes change in inventories at the current unit cost but not the revaluation of stocks of inventories.

$$Y(t)=S(t)+\frac{d\,\mathrm{in}(t)}{dt}\cdot UC(t) \tag{28}$$

4.5.2 The labour market

Real wage aspirations function has been originally defined by Godley and Lavoie (2007) with the same unemployment rate sensitivity in the deflation and inflation regions. This formula has been modified as the wage rate appears to be sticky-down. With the original sensitivity the rate of inflation would become negative with the unemployment rate approaching 10%, as experienced during the GFC (as illustrated in Figure 2 and Figure 8). The current formula uses a piecewise-defined function based on equations (G&L 11.18) and (G&L 11.20). In order to make the wage aspirations function less sensitive to the employment rate in the low than in the high employment rate region, it is assumed that $\omega_1 < \omega_2$ (as depicted in Figure 23 and explained in Section 4.1.3).

Real wage aspirations to productivity ratio ω_r

$$\omega_r(t)=\begin{cases}\omega_0+\omega_2\cdot[ER(t)-ER_{max}], if\ ER(t)\geq ER_{max}\\ \omega_0, if\ ER_{max}>ER(t)\geq ER_{min}\\ \omega_0-\omega_1\cdot[ER_{min}-ER(t)], if\ ER(t)<ER_{pr0}\end{cases} \tag{29}$$

Real wage aspirations ω_t

$$\omega_t(t)=\omega_r(t)\cdot pr(t) \tag{30}$$

Employment rate *ER*

The rate of employment is defined as the ratio of labour demand to labour supply. “LS” (labour supply) is defined as the amount of simple labour supplied at the point of full employment and depends on the size of the working-age population and the participation rate, which is exogenous in the model. If more labour than “LS” is demanded, households will still supply it (for example by working overtime) but real wage aspirations start growing, which leads to a distributional conflict and higher inflation. “LD” (labour demand) is defined as the amount of simple labour demanded by the firms.

$$ER(t)=\frac{LD(t)}{LS(t)} \tag{31}$$

Unemployment rate *UR*

$$UR(t)=\begin{cases}1-ER(t), if\ ER(t)<1\\ 0, if\ ER(t)\geq 1\end{cases} \tag{32}$$

Total population *N*

Total population is a state variable with a constant rate of growth (in the model).

$$\frac{dN(t)}{dt}=N(t)\cdot\log[1+gr_N] \tag{33}$$

Labour supply *LS*

The labour force to total population ratio “LF_N” is defined as the exogenous parameter used in the determination of the labour force for the given size of the population.

$$LS(t)=N(t)\cdot LF_N \tag{34}$$

Nominal wage rate *W* (G&L 11.21)

The wage rate is a state variable. The rate of change of the variable depends on the discrepancy between the target and current values. This differential equation defines a lagged adjustment process.

$$\frac{dW(t)}{dt}=\Omega_3\cdot[\omega_T(t)\cdot p(t)-W(t)] \tag{35}$$

Productivity growth gr_{pr}

The rate of productivity growth is negatively affected by the falling employment rate, which is a proxy variable for falling aggregate demand. It becomes zero in a deep recession, when losses of human capital and low capital investment of firms lead to stagnation of productivity.

$$gr_{pr}(t)=\begin{cases} gr_{pr0}, if\ ER(t)\geq ER_{pr1} \\ gr_{pr0}\cdot\dfrac{ER(t)-ER_{pr0}}{ER_{pr1}-ER_{pr0}}, if\ ER_{pr1}>ER(t)\geq ER_{pr0} \\ 0, if\ ER(t)<ER_{pr0} \end{cases} \quad (36)$$

Labour productivity *pr* (G&L 11.22)

The growth in labour productivity is a cumulative process described by a differential equation and labour productivity is a state variable.

$$\frac{d\,pr(t)}{dt}=pr(t)\cdot\log[1+gr_{pr}(t)] \quad (37)$$

Target labour demand *LD_T* (G&L 11.23)

This variable replaces "N_T", defined by Godley and Lavoie (2007) as the desired employment. It is denominated in [workers/year]. Target labour demand is the demand for simple and complex labour, reduced "at costs" to simple labour. Labour is supplied in the model by both low-income and high-income households. High-income households supply more expensive complex labour, while low-income households supply simple labour. The ratio between complex and simple labour in the economy may change in the long run but is inelastic in the short run.

$$LD_T(t)=\frac{y(t)}{pr(t)} \quad (38)$$

Actual labour demand *LD* (G&L 11.24, modified)

The adjustment of the actual labour demand to the target labour demand is a lagged process. Time constants defining employment adjustment during expansion and recession are different as hiring usually takes more time than sacking workers when companies go bankrupt. η_{LD} is the reciprocal of the time constant describing the net hiring process when the economy is growing while η_{LDr} is the reciprocal of the time constant describing the net reduction of level of employment process during a recession. We assume that η_{LDr} is significantly larger than η_{LD}.

$$\frac{d\,LD(t)}{dt}=\begin{cases} \eta_{LD}\cdot[LD_T(t)-LD(t)], if\ LD_T(t)-LD(t)>0 \\ \eta_{LDr}\cdot[LD_T(t)-LD(t)], if\ LD_T(t)-LD(t)\leq 0 \end{cases} \quad (39)$$

Total nominal wage bill *WB* (G&L 11.25)

Workers are paid for the labour time, not for the physical amount of labour they perform.

$$WB(t)=LD(t)\cdot W(t) \tag{40}$$

Nominal wage bill, high income households *WB_h*

It is assumed that a fraction "HILD" of the total wage bill "WB" is paid to high-income households which are supplying complex labour.

$$WB_h(t)=WB(t)\cdot HILD \tag{41}$$

Nominal wage bill, low income households *WB_l*

$$WB_l(t)=WB(t)\cdot(1-HILD) \tag{42}$$

Labour unit cost *UC* (G&L 11.26)

The cost of direct labour, reduced to simple labour, embedded in the physical unit of the product, averaged for the whole economy. It may differ form the normal labour unit cost as the wage bill is affected by the lag in the adjustment of the labour force. It is equal to normal labour unit cost when target labour demand is equal to the actual labour demand.

$$UC(t)=\frac{WB(t)}{y(t)} \tag{43}$$

Normal labour unit cost *NUC* (G&L11.27)

The cost of direct labour, reduced to simple labour, embedded in a physical unit of the product. Normal labour unit cost is only determined by the wage rate and labour productivity.

$$NUC(t)=\frac{W(t)}{pr(t)} \tag{44}$$

4.5.3 Prices, markups and profits

Normal cost pricing *p* (G&L 11.29)

Unit price of the single commodity produced in the model.

$$p(t)=[1+\phi(t)]\cdot NUC(t) \tag{45}$$

Actual markup φ (G&L 11.30)

Actual markup implemented as a state variable, the differential equation defines a lagged adjustment process. The value of actual markup adjusts to the value of the markup target.

$$\frac{d\phi(t)}{dt}=\epsilon_M\cdot[\phi_T(t)-\phi(t)] \tag{46}$$

Markup target φ_T (G&L 11.31, modified)

The target markup needs to cover not only profits but also interests on loans and corporate taxes. Expected sales and unit costs are used to estimate expected historic labour costs.

$$\phi_T(t)=\frac{F_{fT}(t)+r_l(t)\cdot L_f(t)+T_f(t)}{s_e(t)\cdot UC(t)} \tag{47}$$

Company tax revenue T_f

Company taxes are levied on realised net profits. Company taxes were introduced into the model to have a more realistic value of the spending multiplier.

$$T_f(t)=F_f(t)\cdot\Theta_f \tag{48}$$

Realised net profits F_f (G&L 11.37, modified)

$$F_f(t)=S(t)-WB(t)-r_l(t)\cdot L_f(t)-T_f(t) \tag{49}$$

Planned gross profits F_{fT} (G&L 11.34, modified)

The planned gross profits (gross profits target) only include distributed profits and retained profits.

$$F_{fT}(t)=FU_{fT}(t)+FD_f(t) \tag{50}$$

Planned retained earnings of firms FU_{fT} (G&L 11.35)

Firms set the target level of retained earnings to finance a fixed fraction of gross corporate investment.

$$FU_{fT}(t)=\psi_U\cdot I_f(t) \tag{51}$$

Dividends of firms FD_f (G&L 11.36)

$$FD_f(t)=\psi_D\cdot F_f(t) \tag{52}$$

Retained earnings of firms FU_f (G&L 11.38, modified)

$$FU_f(t)=F_f(t)-FD_f(t) \tag{53}$$

Loans to firms Lf (G&L 11.39)

The stock of firms debt is a state variable. Firms take loans from banks to cover their investment needs not funded from retained earnings and new equities sales. For simplicity, no defaults are assumed on the corporate debt.

$$\frac{dL_f(t)}{dt}=I_f(t)-FU_f(t)-NES(t) \tag{54}$$

Quantity of equities e

Quantity of equities is a stock variable.

$$\frac{de(t)}{dt}=\frac{NES(t)}{p_e(t)} \tag{55}$$

Value of new equity sales NES (G&L 11.41, modified)

A fixed fraction of gross corporate investment is financed by new equity sales.

$$NES(t)=\psi_N\cdot I_f(t) \tag{56}$$

Firms dividend yield r_K (G&L 11.42)

$$r_K(t)=\frac{FD_f(t)}{e(t)\cdot p_e(t)} \tag{57}$$

Price to earnings ratio PE (G&L 11.43)

$$PE(t)=\frac{e(t)\cdot p_e(t)}{F_f(t)} \tag{58}$$

Tobin's q ratio (G&L 11.44)

$$q(t)=\frac{e(t)\cdot p_e(t)+L_f(t)}{K(t)+\mathrm{IN}(t)} \quad (59)$$

4.5.4 Households – income, consumption and financial wealth

Nominal gross personal income of high-income households YP_h (G&L 11.45, modified).

The stock of deposits held by high-income households is equal to the total stock of deposits minus the stock of deposits held by the foreign sector.

$$YP_h(t)=WB_h(t)+FD_f(t)+FD_b(t)+r_m\cdot[M(t)-M_{FS}(t)]+r_m\cdot B_h(t) \quad (60)$$

Nominal gross personal income of low-income households YP_l

The only source of gross personal income of low-income households are wages.

$$YP_l(t)=WB_l(t) \quad (61)$$

Total nominal gross personal income YP

$$YP(t)=YP_h(t)+YP_l(t) \quad (62)$$

Income taxes, low-income households T_{hl} (G&L 11.46, modified)

Low and high income households pay taxes on their nominal gross personal income. The taxes are determined using different tax rates, Θ_l and Θh. For simplicity, labour and capital income is taxed at the same rate. Adding a more complex taxation schedule to the model (such as taxing capital income at a different rate) would not alter the fundamental functional behaviour of the taxation system, which reduces the disposable income available to each of the social classes at a different ratio and partially offsets the government expenditures.

$$T_{hl}(t)=\Theta_l\cdot YP_l(t) \quad (63)$$

Income taxes, high-income households T_{hh}

$$T_{hh}(t)=\Theta_h\cdot YP_h(t) \quad (64)$$

Total income taxes T

$$T_h(t)=T_{hh}(t)+T_{hl}(t) \quad (65)$$

Nominal disposable income, high-income households YD_{rh} (G&L11.47)

$$YD_{rh}(t)=YP_h(t)-T_{hh}(t)-r_l(t)\cdot L_{hh}(t) \quad (66)$$

Nominal disposable income, low-income households YD_{rl}

The nominal disposable income of low-income households also includes unemployment benefits (social transfer payments). This income is not taxed in the model.

$$YD_{rl}(t)=YP_l(t)-T_{hl}(t)-r_l(t)\cdot L_{hl}(t)+UB(t) \quad (67)$$

Unemployment benefits UB

Unemployment benefits are considered to be a proxy for all social transfer payments flowing to low-income households.

$$UB(t)=UR(t)\cdot LS(t)\cdot UBR\cdot W(t) \quad (68)$$

Total nominal disposable income YD_r

$$YD_r(t)=YD_{rh}(t)+YD_{rl}(t) \quad (69)$$

Real disposable income, high-income households yd_{rh}

$$yd_{rh}(t)=\frac{YD_{rh}(t)}{p(t)} \quad (70)$$

Real disposable income, low-income households yd_{rl}

$$yd_{rl}(t)=\frac{YD_{rl}(t)}{p(t)} \quad (71)$$

Total real disposable income yd_r

$$yd_r(t)=yd_{rh}(t)+yd_{rl}(t) \quad (72)$$

Stock of broad money held by high-income households V_{Mh}

The stock of broad money held by high-income households is a state variable. It includes various forms of money and near-money such as currency, deposits and treasury debt securities (bills). Since the simplified model does not include long-term government securities (bonds), there is no revaluation of assets when the interest rates change.

$$\frac{dV_{Mh}(t)}{dt} = YD_{rh}(t) - C_h(t) - I_{hh}(t) + NL_h(t) - NES(t) \tag{73}$$

Stock of broad money held by low-income households V_{Ml}

The stock of broad money held by low-income households is also a state variable. It only includes currency held for "transactions motive" as it is assumed that low-income households do not have savings in banks and do not purchase treasury debt securities. This assumption is necessary because for simplicity the model contains only two social classes, not a broad spectrum of income groups. If the aggregate consumption function of the low-income group is modified by lowering the marginal propensity to consume, it is difficult to get a realistic value of the fiscal spending multiplier.

$$\frac{dV_{Ml}(t)}{dt} = YD_{rl}(t) - C_l(t) - I_{hl}(t) + NL_l(t) \tag{74}$$

Total wealth of households stored in broad money V_M

$$V_M(t) = V_{Mh}(t) + V_{Ml}(t) \tag{75}$$

Financial market asset position of high-income households V_{fmah}

Only high-income households have "investible wealth" (a greater than zero financial market asset position). The financial market asset position does not include the stock of currency "H_{hh}" held for "transactions motive", which cannot be invested.

$$V_{fmah}(t) = V_{Mh}(t) - H_{hh}(t) + e(t) \cdot p_e(t) \tag{76}$$

Nominal wealth target of low-income households V_{MlT}

$$V_{MlT}(t) = \lambda_c \cdot C_l(t) \tag{77}$$

Nominal net wealth, high-income households V_h

The nominal net wealth of high-income households includes the financial wealth position, the own capital of banks and the value of housing assets. Mortgage debt is subtracted from the net wealth.

$$V_h(t)=V_{Mh}(t)+e(t)\cdot p_e(t)+OF(t)-L_{hh}(t)+V_{REh}(t) \tag{78}$$

Nominal net wealth, low-income households V_l

The nominal net wealth of low-income households includes the financial wealth position (consisting of cash) and the value of housing assets. Mortgage debt is subtracted from the net wealth.

$$V_l(t)=V_{Ml}(t)+V_{REl}(t)-L_{hl}(t) \tag{79}$$

Total net nominal wealth of households V

$$V(t)=V_h(t)+V_l(t) \tag{80}$$

Real net wealth, high-income households v_h (G&L 11.51)

$$v_h(t)=\frac{V_h(t)}{p(t)} \tag{81}$$

Real net wealth, low-income households v_l

$$v_l(t)=\frac{V_l(t)}{p(t)} \tag{82}$$

Total net real wealth v

$$v(t)=v_h(t)+v_l(t) \tag{83}$$

Expected real net wealth of high-income households v_{he}

The expected real net wealth of high-income households is determined using a lagged adaptive expectations formula. Trend growth ratio is known as it is determined by productivity growth and population growth. Net wealth expectations adjust more rapidly in a recession than when the wealth is growing, it is assumed that $\varepsilon_{vh}<\varepsilon_{vhr}$.

$$\frac{v_{he}(t)}{dt}=\begin{cases} v_{he}(t)\cdot\log\{(1+gr_N)\cdot[1+gr_{pr}(t)]\}+\epsilon_{vh}\cdot[v_h(t)-v_{he}(t)], if\ v_h(t)\geq v_{he}(t) \\ v_{he}(t)\cdot\log\{(1+gr_N)\cdot[1+gr_{pr}(t)]\}+\epsilon_{vhr}\cdot[v_h(t)-v_{he}(t)], if\ v_h(t)<v_{he}(t) \end{cases} \tag{84}$$

Real consumption of high-income households c_h (G&L 11.53, modified)

High-income households have zero spending propensity out of disposable income. Their consumption depends only on their expected net wealth.

$$c_h(t) = \alpha_2 \cdot v_{he}(t) \quad (85)$$

Real consumption of low-income households c_l

$$c_l(t) = \frac{C_l(t)}{p(t)} \quad (86)$$

Total real consumption c

$$c(t) = c_h(t) + c_l(t) \quad (87)$$

Real consumption of domestically manufactured products and services cd

$$cd(t) = (1-\mu) \cdot c(t) \quad (88)$$

Nominal consumption of high-income households C_h (G&L 11.52)

$$C_h(t) = p(t) \cdot c_h(t) \quad (89)$$

Nominal consumption of low-income households C_l

Nominal consumption expenditures are equal to the disposable income minus the net saving in currency (“transactions motive”). The fraction of the net mortgage borrowing spent on the purchases of land from previous owners is added to the disposable income.

$$C_l(t) = YD_{rl}(t) + NL_l(t) - I_{hl}(t) - \epsilon \cdot [V_{MIT}(t) - V_{MI}(t)] \quad (90)$$

Nominal consumption C

The consumption function has the marginal propensity to consume, determined by the ratio of disposable income of low-income households to total disposable income. The fraction of the net mortgage borrowing spent on the purchases of land from previous owners is added to the disposable income.

$$C(t) = C_h(t) + C_l(t) \quad (91)$$

Nominal consumption of domestically manufactured products and services *CD*

$$CD(t)=(1-\mu)\cdot C(t) \tag{92}$$

Amount of cash held by high-income households H_{hh} (G&L 11.69)

$$H_{hh}(t)=\lambda_c\cdot C_h(t) \tag{93}$$

Amount of cash held by low-income households H_{hl}

$$H_{hl}(t)=V_{Ml}(t) \tag{94}$$

Amount of cash held by households H_h

$$H_h(t)=H_{hh}(t)+H_{hl}(t) \tag{95}$$

Government securities held by households B_h (G&L 11.64)

For simplicity only short term government securities (bills) are included in the model.

$$B_h(t)=V_{fmah}(t)\cdot[\lambda_{20}+\lambda_{22}\cdot r_b-\lambda_{24}\cdot r_K(t)-\frac{\lambda_{25}\cdot YD_{rh}(t)}{V_h(t)}] \tag{96}$$

Price of equities p_e (G&L 11.66)

The quantity of equities is determined by a separate process of new equity sales, described by equation (55).

$$p_e(t)=\frac{V_{fmah}(t)}{e(t)}\cdot[\lambda_{40}-\lambda_{42}\cdot r_b+\lambda_{44}\cdot r_K(t)-\frac{\lambda_{45}\cdot YD_{rh}(t)}{V_h(t)}] \tag{97}$$

Money deposits *M*

Money deposits of high-income households are a residual element remaining after allocation of fractions of the stock of broad money to government securities and cash. Foreign sector deposits count towards the total stock of money deposits in banks.

$$M(t)=V_{Mh}(t)-B_h(t)-H_{hh}(t)+M_{FS}(t) \tag{98}$$

4.5.5 Households – mortgage lending and real estate assets

Total gross mortgage lending to households *GL*

The model assumes that all loans to households are mortgages and are spent on real estate assets. The ratio of gross household lending to disposable income "η" is determined exogenously.

$$GL(t) = \eta \cdot YD_r(t) \tag{99}$$

Gross mortgage lending to high-income households GL_h

A fixed fraction "HILE" of the total gross lending are loans to high-income households.

$$GL_h(t) = HILE \cdot GL(t) \tag{100}$$

Gross mortgage lending to low-income households GL_l

$$GL_l(t) = GL(t) - GL_h(t) \tag{101}$$

Mortgage loans repayment by high-income households REP_h

$$REP_h(t) = \delta_{rep} \cdot L_{hh}(t) \tag{102}$$

Mortgage loans repayment by low-income households REP_l

$$REP_l(t) = \delta_{rep} \cdot L_{hl}(t) \tag{103}$$

Total mortgage loans repayment REP (G&L 11.59)

$$REP(t) = REP_h(t) + REP_l(t) \tag{104}$$

Net mortgage lending to high-income households NL_h

$$NL_h(t) = GL_h(t) - REP_h(t) \tag{105}$$

Net mortgage lending to high-income households NL_l

$$NL_l(t) = GL_l(t) - REP_l(t) \tag{106}$$

Total net mortgage lending NL (G&L 11.58)

$$NL(t) = NL_h(t) + NL_l(t) \tag{107}$$

High-income households mortgage loans L_{hh}

Stock of high-income households mortgage loans is a state variable.

$$\frac{d L_{hh}(t)}{dt} = NL_h(t) \qquad (108)$$

Low-income households mortgage loans L_{hl}

Stock of low-income households mortgage loans is a state variable.

$$\frac{d L_{hl}(t)}{dt} = NL_l(t) \qquad (109)$$

Total mortgage loans L_h (G&L 11.60)

$$L_h(t) = L_{hh}(t) + L_{hl}(t) \qquad (110)$$

Real mortgage lending to high-income households nl_h

$$nl_h(t) = \frac{NL_h(t)}{p(t)} \qquad (111)$$

Real mortgage lending to low-income households nl_l

$$nl_l(t) = \frac{NL_l(t)}{p(t)} \qquad (112)$$

Total real mortgage lending nl (G&L 11.61)

$$nl(t) = nl_h(t) + nl_l(t) \qquad (113)$$

Nominal housing investment by high-income households I_{hh}

It is assumed that a fixed fraction "IHMOV" of the gross lending is spent on construction and contributes towards total investment. The rest of the gross lending is spent on buying land and houses from previous owners. This fraction of new lending is directly added to the disposable income of the corresponding social group (the same value "IHMOV" applies to both income groups) as defined in equation (66).

$$I_{hh}(t) = GL_h(t) \cdot IHMOV \qquad (114)$$

Nominal housing investment by low-income households I_{hl}

$$I_{hl}(t) = GL_l(t) \cdot IHMOV \qquad (115)$$

Total nominal housing investment I_h

$$I_h(t)=I_{hh}(t)+I_{hl}(t) \tag{116}$$

Real housing investment by high-income households i_{hh}

$$i_{hh}(t)=\frac{I_{hh}(t)}{p(t)} \tag{117}$$

Real housing investment by low-income households i_{hl}

$$i_{hl}(t)=\frac{I_{hl}(t)}{p(t)} \tag{118}$$

Total real housing investment i_h

$$i_h(t)=i_{hh}(t)+i_{hl}(t) \tag{119}$$

Nominal value of real estate owned by high-income households V_{REh}

It is assumed that the ratio of the value of real estate to the total value of the mortgages remains nearly constant in the short run. The “REMOR” coefficient is a slowly-changing exogenous parameter.

$$V_{REh}(t)=L_{hh}(t)\cdot REMOR \tag{120}$$

Nominal value of real estate owned by low-income households V_{REl}

$$V_{REl}(t)=L_{hl}(t)\cdot REMOR \tag{121}$$

Total nominal value of real estate V_{RE}

$$V_{\mathrm{RE}}(t)=V_{REh}(t)+V_{REl}(t) \tag{122}$$

Real value of real estate owned by high-income households v_{REh}

$$v_{REh}(t)=\frac{V_{REh}(t)}{p(t)} \tag{123}$$

Real value of real estate owned by low-income households v_{REl}

$$v_{REI}(t)=\frac{V_{REI}(t)}{p(t)} \tag{124}$$

Total real value of real estate v_{RE}

$$v_{\mathrm{RE}}(t)=\frac{V_{\mathrm{RE}}(t)}{p(t)} \tag{125}$$

Total real value of residential buildings v_{RES}

Total real value of residential buildings is defined as equal to the accumulated real investment (the spending on construction of buildings) minus the depreciation.

$$\frac{d\,v_{RES}(t)}{dt}=i_h(t)-v_{RES}(t)\cdot\delta_{RES} \tag{126}$$

Total nominal value of residential buildings V_{RES}

$$V_{RES}(t)=v_{RES}(t)\cdot p(t) \tag{127}$$

4.5.6 The public sector

Taxes T

It is assumed for simplicity that taxes are only levied on household income and corporate profits.

$$T(t)=T_{hh}(t)+T_{hl}(t)+T_f(t) \tag{128}$$

Nominal government expenditures G

Government expenditures are spent on purchases of goods and services from firms. Underemployment benefits are not counted as expenditures as they are accounted for in disposable income of low-income households.

$$G(t)=p(t)\cdot g(t) \tag{129}$$

Real government expenditures g

The volume of real government expenditures is a state variable, in some simulation scenarios the ratio of government expenditures to the GDP (“GYR”) is a control variable.

$$\frac{d\,g(t)}{dt} = g(t) \cdot \log\{1 + gr_g - \epsilon_{GYR} \cdot [\frac{g(t)}{y} - GYR]\} \quad (130)$$

Nominal government deficit *PSBR* (G&L 11.73, modified)

$$PSBR(t) = G(t) + r_m \cdot [B_h(t) + B_b(t) + B_{FS}(t)] - T(t) + UB(t) \quad (131)$$

Government securities *B*

For simplicity it is assumed that all government securities are short-term treasury bills.

$$\frac{d\,B(t)}{dt} = PSBR(t) \quad (132)$$

Nominal government debt *GD* (G&L 11.75)

$$GD(t) = B_b(t) + B_h(t) + H(t) + B_{FS}(t) \quad (133)$$

High powered money *H* (G&L 11.76)

$$H(t) = H_b(t) + H_h(t) \quad (134)$$

Government securities held by central bank B_{cb} (11.82)

$$B_{cb}(t) = H(t) \quad (135)$$

4.5.7 The foreign sector

Real net imports *im*

$$\mathrm{im}(t) = c(t) \cdot \mu \quad (136)$$

Nominal net imports *IM*

It is assumed that import prices expressed in the local currency are the same as for domestic products.

$$\mathrm{IM}(t) = C(t) \cdot \mu \quad (137)$$

Nominal net wealth of the foreign sector V_{FS}

Nominal net wealth of the foreign sector is the net value of the assets of the foreign sector included on the balance sheets of the banks and the government sector.

$$\frac{dV_{FS}(t)}{dt} = \mathrm{IM}(t) + r_m \cdot B_{FS}(t) + r_m \cdot M_{FS}(t) \qquad (138)$$

Government securities owned by the foreign sector B_{FS}

$$B_{FS}(t) = V_{FS}(t) \cdot \lambda_{50} \qquad (139)$$

Bank deposits of the foreign sector M_{FS}

$$M_{FS}(t) = V_{FS}(t) \cdot (1 - \lambda_{50}) \qquad (140)$$

4.5.8 The banking sector

Retained earnings FU_b (G&L 11.107)

$$FU_b(t) = F_b(t) - FD_b(t) \qquad (141)$$

Own funds OF (G&L 11.108)

Own funds of banks are the capital (equity) of the banking sector.

$$\frac{dOF(t)}{dt} = FU_b(t) \qquad (142)$$

Bills held by banks B_b (G&L 11.91)

Banks hold all the government securities which are not held by households, the central bank and the foreign sector.

$$B_b(t) = B(t) - B_h(t) - B_{cb}(t) - B_{FS}(t) \qquad (143)$$

Realised profits of banks F_b (G&L 11.105)

$$F_b(t) = r_l(t) \cdot [L_f(t) + L_h(t)] + r_m \cdot B_b(t) - r_m \cdot M(t) \qquad (144)$$

Bank reserves H_b (G&L 11.90)

$$H_b(t) = \rho \cdot M(t) \qquad (145)$$

Rate of interest on bank loans r_l (G&L 11.98)

$$r_l(t) = r_m + add_l(t) \qquad (146)$$

Bank own funds target OF_T (G&L 11.99)

$$OF_T(t)=NCAR\cdot[L_f(t)+L_h(t)] \tag{147}$$

Target retained earnings of banks FU_{bT} (G&L 11.101)

$$FU_{bT}(t)=R[\frac{dOF_T(t)}{dt}] \tag{148}$$

Bank dividends FD_b

It is assumed that the rate of return on capital is the same as in the corporate sector. This differs from what has been assumed by Godley and Lavoie (2007)

$$FD_b(t)=r_K(t)\cdot OF(t) \tag{149}$$

Target profits of banks F_{bT} (G&L 11.104)

$$F_{bT}(t)=FD_b(t)+FU_{bT}(t) \tag{150}$$

Spread of the lending rate over the deposit rate add_l (G&L 11.106, modified)

$$add_l(t)=\frac{F_{bT}(t)-r_m\cdot B_b(t)+r_m\cdot[M(t)-L_f(t)-L_h(t)]}{L_f(t)+L_h(t)} \tag{151}$$

Capital adequacy ratio CAR (G&L 11.109)

$$CAR(t)=\frac{OF(t)}{L_f(t)+L_h(t)} \tag{152}$$

4.5.9 Ensuring accounting and stock-flow consistency of the model

The concept of internal consistency of SFC models has been mentioned in Section 3.2. Conditions defined below are validated using “assert” statements. Additionally “assert” statements are executed for individual variables to ensure that their values are sensible from an economic point of view (for example, negative prices are not allowed and if they appear, the simulation terminates with an error).

All three measures of nominal GDP defined below need to be equal.

$$Y(t)=Y_{\exp}(t) \tag{153}$$

$$Y(t)=Y_{inc}(t) \tag{154}$$

The value of GDP obtained using the production (value added) approach is the "Y" calculated in the model. The value obtained using the expenditure approach "Y_{exp}" is defined as:

$$Y_{\exp}(t)=S(t)+CD(t)+I(t) \tag{155}$$

The value obtained using the income approach "Y_{inc}" is defined as:

$$Y_{inc}(t)=W(t)+F_b(t)+F_f(t)+L_f(t)\cdot r_l(t) \tag{156}$$

Bank assets have to be equal to the sum of bank liabilities and equity.

$$\mathrm{LE}_B(t)=A_B(t) \tag{157}$$

The sum of bank assets "A_B" is defined as:

$$A_B(t)=L_f(t)+L_h(t)+B_b(t)+H_B(t) \tag{158}$$

The sum of bank liabilities and equity "LE_B" is defined as:

$$\mathrm{LE}_B(t)=OF(t)+M(t) \tag{159}$$

Money demand and money supply must be equal.

$$M_d(t)=M_s(t) \tag{160}$$

Money demand "M_d" is defined as the quantity of money deposits in the model. The domestic component is the residual from the stock of broad money held by households after subtracting government securities and currency demanded by the households (in the model, only high-income households have deposits). The stock of broad money consists of the accrued household net income and transfers, minus all spending. The foreign component of the money demand (money deposits belonging to the foreign sector) also needs to be included in the determination of the total money demand.

$$M_d(t)=V_M(t)+M_{FS}(t)-B_h(t)-H_h(t) \tag{161}$$

The supply of deposits "M_s" is determined by the accounting identity applied to the balance sheet of the banking sector. The growth of the stock of banks' own capital is a result of the accrual of net profits minus the bank dividends. It is assumed in the model that firms do not have bank deposits. This condition validates the "stock-flow" consistency.

$$M_s(t)=L_f(t)+L_h(t)+B_b(t)+H_b(t)-OF(t) \tag{162}$$

Money demand "M_d" (the quantity of bank deposits) can also be calculated from the stock of broad money which is held in the model by high-income households. The demand calculated with this method must be equal to the value calculated from the stock of broad money held by all households.

$$M_d(t)=M(t) \tag{163}$$

The stock of broad money must be equal to the stock of government securities held by the private sector and bank assets minus the bank capital.

$$V_M(t)+V_{FS}(t)=\mathrm{LE}_{GB}(t) \tag{164}$$

The stock of broad money "V_M" held by households is defined as the sum of deposits, government securities and currency. The stock of broad money "V_{FS}" held by the foreign sector consists of bank deposits and government securities. For simplicity only short-term government securities (bills) are defined in the model. No capital gains or losses on long-term securities need to be accounted for. The stock of government securities and bank assets minus the bank capital "LE_{GB}" consists of all the government securities (including these which are held by the central bank) and the sum of bank loans minus the own funds of the banking sector. The value of the government securities held by the central bank in the model is equal to the amount of the high-powered money held by the households and the banks.

$$\mathrm{LE}_{GB}(t)=B(t)+L_f(t)+L_h(t)-OF(t) \tag{165}$$

4.5.10 The Transaction Flow Matrix of the model

The flow consistency of the model can be validated by checking the sums of rows and columns in a Transaction Flow Matrix as presented by Godley and Lavoie (2007). If the model is flow-consistent, these sums should all be equal to zero. For simplicity, current and capital accounts have been aggregated in the matrix. The sectors and the simplified structure of the monetary flows are depicted in Figure 21

Table 9: Transaction Flow Matrix of the model

Flow	**Firms**	**HH-H**	**HH-L**	**Banks**	**CB**	**Gov.**	**Foreign**	$\sum$
Consumption	C-IM	$-C_h$	$-C_l$				IM	0
Investment	I_h	$-I_{hh}$	$-I_{hl}$					0
Gov. Exp.	G					-G		0
Taxes	$-T_f$	$-T_{hh}$	$-T_{hl}$			T		0
Wages	-WB	WB_h	WB_l					0
Dividends	$-FD_f$	FD_f+FD_b		$-FD_b$				0
Unemp. ben.			UB			-UB		0
Int. on loans	$-r_l \cdot L_f$	$-r_l \cdot L_{hh}$	$-r_l \cdot L_{hl}$	$r_l \cdot (L_h+L_f)$				0
Int. on dep.		$r_m \cdot (M-M_{FS})$		$-r_m \cdot M$			$r_m \cdot M_{FS}$	0
Int. on bills		$r_m \cdot B_h$		$r_m \cdot B_b$		$-r_m \cdot (B-B_{cb})$	$r_m \cdot B_{FS}$	0
Net lending	$d\,L_f/dt$	$d\,L_{hh}/dt$	$d\,L_{hl}/dt$	$-d\,L/dt$				0
Net saving (M)		$-d(M-M_{FS})/dt$		$d\,M/dt$			$-d\,M_{FS}/dt$	0
Net saving (H)		$-d\,H_{hh}/dt$	$-d\,H_{hl}/dt$	$-d\,H_b/dt$	$d\,H/dt$			0
Net saving (B)		$-d\,B_h/dt$		$-d\,B_b/dt$	$-d\,B_{cb}/dt$	$d\,B/dt$	$-d\,B_{FS}/dt$	0
Net eq. sales	$d\,e/dt \cdot p_e$	$-d\,e/dt \cdot p_e$						0
$\sum$	0	0	0	0	0	0	0	0

The zero sum condition on these rows of the matrix, where it is not self-evident that the elements add up to zero, has been validated by inspecting the following equations:

- Consumption flows - (92), (91) and (137),
- Investment flows - (116),
- Taxes - (128),
- Wages - (41) and (42),
- Interests on loans - (110),

- Interests on bills paid to non-government sector (for simplicity, it is assumed that the Treasury does not pay interests to the Central Bank as these interests are returned back to the Government) - (131) and (143),
- Net saving in currency (H) -(95) and (134),
- Net saving in bills (government securities) - (143)

The zero sum conditions on the columns of the matrix, corresponding to budget constraints on all the sectors, cannot be validated analytically without solving the model. These conditions will be validated using the values obtained from the numerical simulation in Section 4.7.3.

4.6 Model calibration in the baseline scenario

The following values were used in model calibration (the procedure has been described in Section 4.1.5)

Table 10: Calibration of the model in "Reference" scenario

Variable or parameter	Symbol	Measured value	Assumed value	Data source
Population	N	2.36e+8	2.36e+8	U.S. Bureau of Economic Analysis (2019), retrieved from FRED as [POPTHM]
Annual population growth	gr_N	0.959%	1%	U.S. Bureau of Economic Analysis (2019), retrieved from FRED as [POPTHM]
Labour force	LS	1.13e+12	1.11e+12	U.S. Bureau of Labor Statistics (2019), Civilian Labor Force, Retrieved from FRED as [CLF16OV]
Unemployment	UR	7.5%	6.0%	U.S. Bureau of Labor Statistics (2019), Civilian Unemployment Rate, Retrieved from FRED as [UNRATE]
GDP	Y	4e+12	4e+12	U.S. Bureau of Economic Analysis (2019) Gross Domestic Product, Retrieved from FRED as [GDP]
Wage Bill	WB	2.215e+12	3.11e+12	U.S. Bureau of Economic Analysis (2019) National income: Compensation of employees, Retrieved from FRED as

				[A033RC1A027NBEA]
Government spending on goods and services to GDP Ratio	G/Y	0.22	0.24	Chinn. M. (2013)
Share of fixed residential investment in the GDP	I_h/Y	4.7%	4.96%	U.S. Bureau of Economic Analysis (2019) Shares of gross domestic product: Gross private domestic investment: Fixed investment: Residential, Retrieved from FRED as [A011RE1Q156NBEA]
Ratio of residential mortgage originations to GDP	GL/Y	5.78%	9.01%	U.S. Department of Housing (1998), U.S. Housing Market Conditions, Table 17. Residential Mortgage Originations by Building Type: 1970-Present, Retrieved from https://www.huduser.gov/periodicals/ushmc/winter97/histdat5.html#tbl4_17
Ratio of total household debt to GDP	L_h/Y	49%	41.6%	Bank of International Settlements (2019), Total credit to households (core debt) as a percentage of GDP. Retrieved from https://stats.bis.org/statx/srs/table/f3.1?p=20183&c=
Ratio of total household wealth to GDP	V/Y	3.84	2.48	Board of Governors of the Federal Reserve System (US) (2019), Households and Nonprofit Organizations; Total Assets, Level Retrieved from FRED as [TABSHNO]
Households home mortgages	L_h	1.20e+12	1.66e+12	Board of Governors of the Federal Reserve System (US) (2019), Households and Nonprofit Organizations; Home Mortgages; Liability, Level Retrieved from FRED as [HHMSDODNS]
Households owners' equity in real estate	$V_{RE} - L_h$	2.76e+12	3.83e+12	Board of Governors of the Federal Reserve System (US) (2019), Households; Owners' Equity in Real Estate, Level, Retrieved from FRED as [OEHRENWBSHNO]
Share of net income of high income	YD_{rh}/YD_r	45%	39%	Semega, Fontenot and Kollar (2018)

households in the total net income				
Share of high income households net wealth in the total household wealth	V_h/V	81.3% (data from 1983)	76%	D'Ambrosio and Wolff (2008)
Total credit to non-financial corporations as a fraction of GDP	L_f/Y	56.3%	32%	Bank of International Settlements (2019), Total Credit to Non-Financial Corporations, Adjusted for Breaks, for United States. Retrieved from FRED as [QUSNAM770A]
Total public debt to GDP ratio	GD/y	38.4%	83%	Federal Reserve Bank of St. Louis and U.S. Office of Management and Budget (2019). Retrieved from FRED as [GFDEGDQ188S]

Notes:

High-income households are defined in the model as the top 20% (the top quintile); low-income households are defined as the bottom 80%.

The ratio of residential mortgage originations to GDP was volatile in the mid-1980s and the value provided for 1984 is not a multi-year average.

The unemployment rate was volatile in the mid-1980s and the value provided for 1984 is not a multi-year average.

It is assumed that residential construction spending equals 55% of the mortgage origination volume (IHMOV=0.55). The supporting data covering the period 1990-2009 is from U.S. Bureau of Economic Analysis (2019) retrieved from FRED as [PRFI] and from Freddie Mac (2009) Freddie Mac Update January 2009.

It is assumed that mortgage repayment rate (δ_{rep}) is equal to 0.15. It is also assumed that the real estate value to mortgage debt stock ratio (REMOR) is equal to 3.3. These values apply to a growing economy not affected by a housing market slump. The value of REMOR is supported by the following data series from FRED: [HNOREMQ027S] and [HMLBSHNO].

Constant parameters of the model (especially the consumption function coefficients) have been adjusted in order to get a plausible value of the spending multiplier, around 1.6 as in Blanchard and Leigh (2014).

Adjustment rate coefficients β, ε, ε_{GYR}, ε_M, ε_{vh}, ε_{vhr}, η_{LD}, η_{LDr}, γ, γ_u, σ_A, σ_N and σ_{se} have been assumed to have plausible values. Some of the dynamic processes described by these coefficients have time constants (equal to the reciprocals of corresponding adjustment rate coefficients) significantly shorter than one year, the shortest which is meaningful in a discrete time model with a standard one year sampling period, such as GROWTH presented by Godley and Lavoie (2007).

Other exogenous parameters not mentioned above (for example δ), have values taken from GROWTH model (Godley & Lavoie, 2007) or assumed to have plausible values. The values of employment rate thresholds such as “ER..” and wage expectations coefficients ω_1 and ω_2 have been adjusted to reproduce the actual trajectories of variables in “HistoricalGFC” scenario.

Table 11: Full list of exogenous parameters and their values in "Reference" scenario

Symbol	**Value**	**Symbol**	**Value**	**Symbol**	**Value**
ER_{max}	0.96	δ_{rep}	0.15	λ_{42}	0.4
ER_{min}	0.935	δ_{RES}	0.02	λ_{44}	0.4
ER_{pr1}	*0.94*	ε	6	λ_{45}	0.2
ER_{pr0}	*0.90*	ε_{GYR}	*1*	λ_{50}	0.4
GYR	*0.23889*	ε_M	0.1	λ_b	0.0153
HILD	*0.40*	ε_{vh}	0.5	λ_c	0.05
HILE	0.4	ε_{vhr}	1	μ	*0.0350*
HIN	0.2	η	*0.14*	ω_0	*0.88*

IHMOV	0.55	η_{LD}	0.6	ω_1	1.0
LF_N	*0.50854*	η_{LDr}	2	ω_2	2.0
NCAR	0.1	γ	6	ψ_D	*0.30*
Ω_3	0.5	γ_u	0.1	ψ_N	0.04
REMOR	*3.3*	gr_N	0.01	ψ_U	0.75
Θ_f	0.05	gr_g	*0.0302*	r_m	*0.04*
Θ_{hh}	*0.35*	gr_{pr0}	0.02	ρ	0.05
Θ_{hl}	*0.1475*	λ_{20}	0.1	σ_A	6
UBR	0.14251	λ_{22}	0.2	σ_N	6
α_2	0.1	λ_{24}	0.2	σ_T	0.2
β	12	λ_{25}	0.2	σ_{se}	6
δ	0.10667	λ_{40}	*0.5*	u_0	*0.85*

Note: parameters changed in other simulation scenarios are in *italics*.

Table 12: Initial values of state variables in all simulation scenarios

Symbol	**Value**	**Symbol**	**Value**	**Symbol**	**Value**
B	3.30735e+12	V_{FS}	2.95484e+12	k	9.10574893e+12
LD	1.14025488e+8	V_{Mh}	2.94351e+12	φ	0.265515
L_f	1.29836e+12	V_{Ml}	7.44108e+10	pr	66770.47013
L_{hh}	6.65301e+11	W	27303.23475	s_e	7.67541476e+12
L_{hl}	9.97951e+11	e	1e+09	v_{he}	1.46102931e+13
N	2.36e+08	g	1.84895139e+12	v_{RES}	7.70241426e+12
OF	2.96162e+11	in	1.52435540e+12		

4.7 Baseline scenario simulation results

4.7.1 The stability of the model in the baseline scenario

The model running the baseline ("Reference") scenario is stable: all the state variables grow exponentially, which generates straight lines in logarithmic scale. This does not fully reflect historic circumstances as the American economy was not in a state of balanced (equilibrium) growth in 1984.

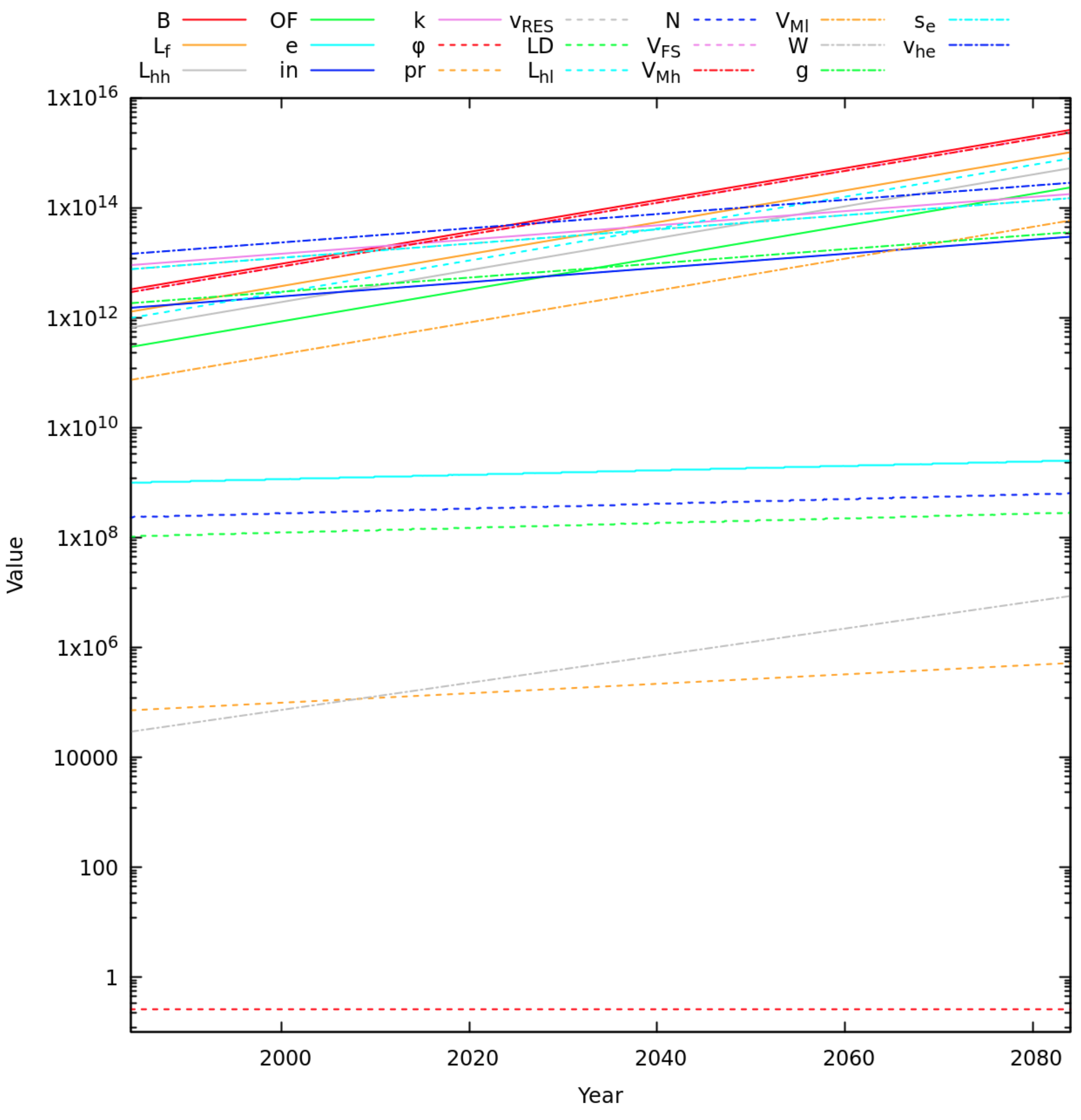

Figure 24: State variables in the baseline scenario

4.7.2 The growth rates of state variables

The growth rates of state variables in the baseline scenario simulation are presented in Table 13. These values have been validated by adding probes to the model. Since the model is defined in continuous time, growth rates calibrated for one year period differ from growth rates defined as logarithmic derivatives of the variables (for example 1% annual growth corresponds to 0.995% growth rate). Several nominal state variables grow at rates depending on endogenous rate of inflation. The quantity of equities also grows at the rate which is endogenous and depends on the price of equities, affected by the portfolio allocation model.

Table 13: Growth rates of state variables

State Variable	**Symbol**	**Growth rate**	**Gr. rate value**
Actual mark-up	φ	0, exogenous	0
Quantity of equities	e	endogenous	0.924%
Population	N	$\log_e(1+gr_N)$, exogenous	0.995%
Labour demand	LD	$\log_e(1+gr_N)$, exogenous	0.995%
Labour productivity	pr	$\log_e(1+gr_{pr})$, exogenous	1.980%
Real capital stock	k	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Real government expenditures	g	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Real value of resid. buildings	v_{RES}	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Real inventory	in	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Expected real sales	s_e	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Exp. real wealth of high-inc. households	v_{he}	$\log_e[(1+gr_{pr})\cdot(1+gr_N)]$, exogenous	2.975%
Wage rate	W	$\pi+\log_e(1+gr_{pr})$, π is endogenous	5.683%
Loans to firms	L_f	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.677%
Capital of banks	OF	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.677%
High-income households' liquid wealth	V_{Mh}	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%
Low-income households' liquid wealth	V_{Ml}	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%
Loans to high-income households	L_{hh}	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%
Loans to low-income households	L_{hl}	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%
Foreign sector nominal wealth	V_{FS}	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%
Government securities	B	$\pi+\log_e[(1+gr_{pr})\cdot(1+gr_N)]$	6.678%

4.7.3 The consistency of the Transaction Flow Matrix of the model

The values of flows calculated at the time equal to 2000.0 have been presented in Table 14. These values have been calculated in a LibreOffice spreadsheet using single-precision floating point numbers.

Table 14: Transaction Flow Matrix of the model (numerical values)

Flow	**Firms**	**HH-H**	**HH-L**	**Banks**	**CB**	**Gov.**	**Foreign**	∑
Consumption	6.35E+12	-2.20E+12	-4.38E+12				2.30E+11	0.00E+00
Investment	5.77E+11	-2.31E+11	-3.46E+11					0.00E+00
Gov. Exp.	2.79E+12					-2.79E+12		0.00E+00
Taxes	-1.11E+11	-1.64E+12	-8.02E+11			2.56E+12		-1.10E-03
Wages	-9.06E+12	3.63E+12	5.44E+12					0.00E+00
Dividends	-6.64E+11	7.35E+11		-7.05E+10				0.00E+00
Unemp. ben.			6.79E+10			-6.79E+10		0.00E+00
Int. on loans	-2.03E+11	-1.04E+11	-1.56E+11	4.64E+11				0.00E+00
Int. on dep.		2.95E+11		-5.02E+11			2.06E+11	0.00E+00
Int. on bills		4.30E+10		1.66E+11		-3.47E+11	1.38E+11	0.00E+00
Net lending	2.52E+11	1.29E+11	1.94E+11	-5.76E+11				0.00E+00
Net saving (M)		-4.93E+11		8.38E+11			-3.44E+11	0.00E+00
Net saving (H)		-7.35E+09	-1.45E+10	-4.19E+10	6.37E+10		-2.30E+11	3.24E-05
Net saving (B)		-7.18E+10		-2.78E+11	-6.37E+10	6.43E+11		0.00E+00
Net eq. sales	7.51E+10	-7.51E+10						0.00E+00
∑	0.00E+00	0.00E+00	0.00E+00	8.85E-04	0.00E+00	0.00E+00	0.00E+00	

The deviations of sums of flows from zero value for firms, banks and high-income households (these have the highest values), evaluated in OpenModelica with double-precision floating point representation are also presented in Figure 25. The deviations from zero values are evidently caused by numerical errors and they are larger for the sums of variables which have higher absolute values.

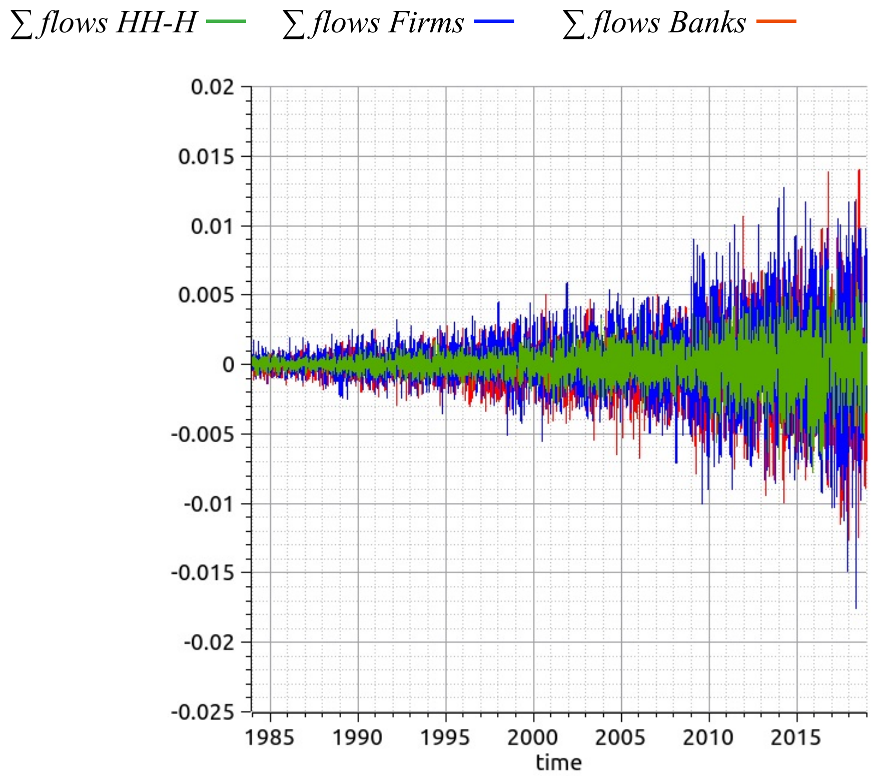

Figure 25: The deviations of sums of flows from zero value

4.8 Summary of the simulation scenarios

The list of the variables shocked in the simulation scenarios is presented in Table 15.

Table 15: Exogenous variables shocked in the simulation scenarios

Simulation scenario	**Shocked variables**	**Remarks**
Reference		Equilibrium growth
FiscalStimulus	gr_g (affecting g)	Small transient positive demand shock
FiscalExperiment	gr_g (affecting g)	Small persistent negative demand shock
ProductivityExperiment	ER_{pr1}(affecting gr_{pr})	Small persistent negative productivity shock
TradeBalanceChanges	μ (affecting im)	Small transients demand shocks
MonetaryStimulus	r_m	Small persistent reduction of the interest rate
DistributionalChanges	HILD (affecting WBh, WBl)	Small persistent change in income distribution towards high-income households
StockmarketBubble	λ_{40}, u_0	Small transient stock market bubble (a positive shock to share prices and firms investment)

HousingBubble	η	Small transient real estate bubble (a positive shock to real estate investment followed by a negative shock)
HousingBubblePriceCrash	η, REMOR	Small transient real estate bubble followed by a price crash
HistoricalNoBubblesNoStimuli	ψ_D, ω_0, HILD, GYR, μ, LF_N, Θ_{hh}	Historical trajectories of the distributional parameters, tax rates and government spending in the long run
HistoricalNoStimuli	ψ_D, ω_0, HILD, GYR, μ, LF_N, Θ_{hh}, η, REMOR	Historical trajectories of the distributional parameters, tax rates and government spending in the long run with the dot and real estate bubbles simulated
HistoricalNoFiscalStimulus	ψ_D, ω_0, HILD, GYR, μ, LF_N, Θ_{hh}, η, REMOR, r_m	As in HistoricalNoStimuli with the historical trajectory of the interest rates
HistoricalGFC	ψ_D, ω_0, HILD, GYR, μ, LF_N, Θ_{hh}, η, REMOR, r_m, gr_g	Historical trajectories of the distributional parameters, tax rates, interest rates and government spending in the long and short run with the dot and real estate bubbles simulated
FiscalExpansion	GYR, ε_{GYR}	Long-run simulation demonstrating increasing the GDP in the long run by increasing the government spending to GDP ratio
IncomeRedistribution	Θ_{hh}, Θ_{hl}	Long-run simulation demonstrating increasing the GDP in the long run by redistributing the disposable income towards low-income households achieved by changing the tax rates

4.9 Dynamic calibration of the model

The list of simulation scenarios has been presented in Section 4.1.4. Except for the "Reference" scenario, all the simulations involve some dynamic processes. Constant parameters of the model need to be adjusted so that the magnitude of dynamic responses of the model to exogenous changes in the aggregate demand is correct.

4.9.1 The supermultiplier in the short run

The series of graphs presented in this section show the changes in the real GDP and corporate investment in the model when real government expenditure is increased temporarily (a "fiscal stimulus"). This corresponds to the scenarios described in Section 4.1.2. The graphs have been generated by re-running the baseline ("Reference") scenario simulation with "g" temporarily increased by 5% (by changing "gr_g", the rate of growth of government expenditures) while "ε_{GYR}" has been set to 0. Changes in the exogenous parameter "g" are depicted in Figure 26.

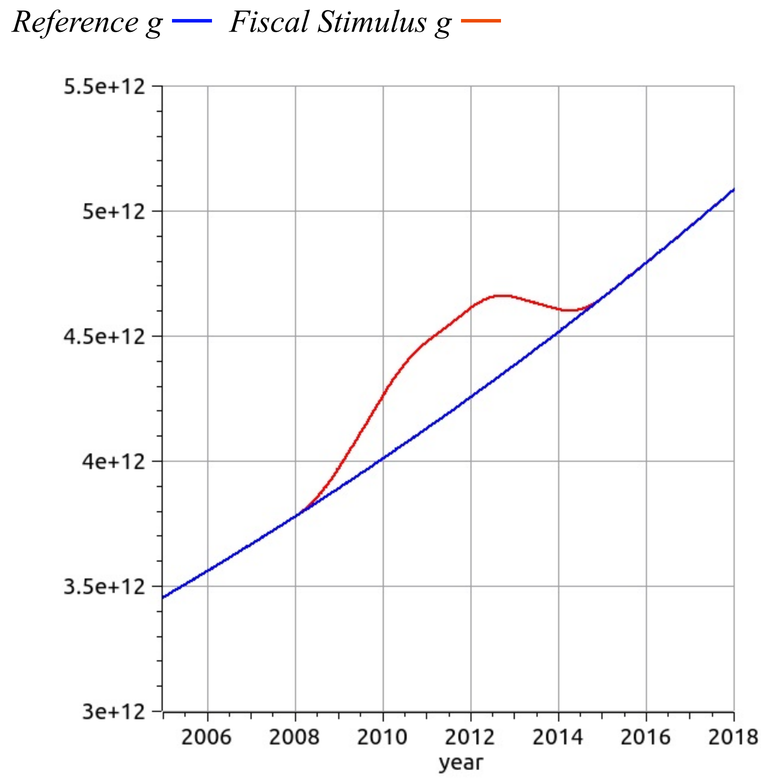

Figure 26: Government consumption expenditures in fiscal stimulus and baseline scenarios.

The ratio of increase of GDP to increase of government spending corresponds to a government spending multiplier. The parameters of the model have been adjusted so that the multiplier is about 1.6 (as mentioned in Section 4.1.5). In fact the multiplier in the model also incorporates the effects of an investment accelerator so it should be called a "supermultiplier". The response of the GDP of the economy to a short fiscal stimulus is shown in Figure 27. It illustrates the process of the amplification of the temporary increase in the autonomous component of the aggregate demand by the economy.

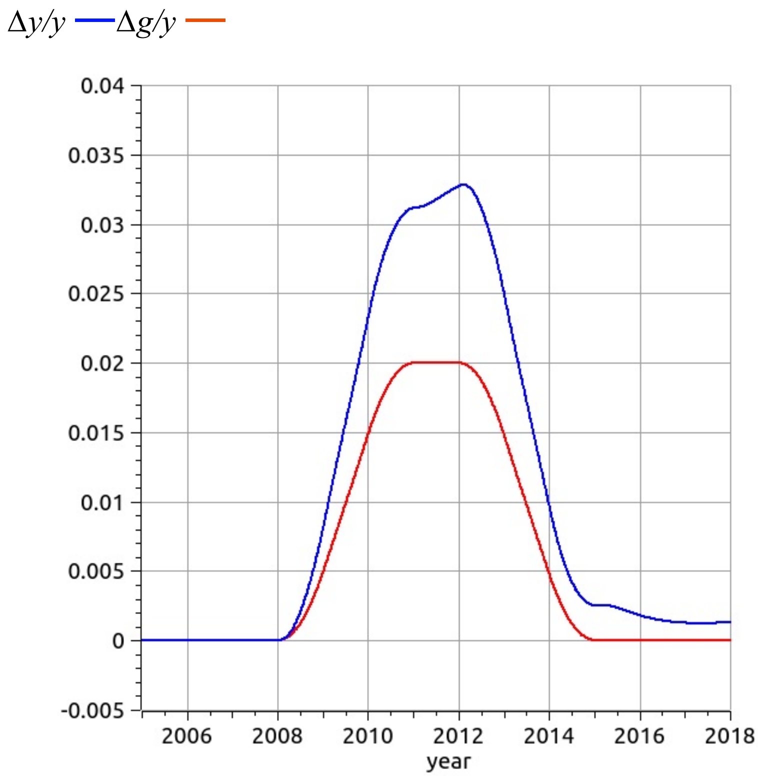

Figure 27: The fiscal supermultiplier

In the model, low-income households have high marginal spending propensity (almost equal to one) while in the short-run the marginal spending propensity of high-income households is almost equal to zero. This is an arbitrary assumption, required to achieve a realistic value of the marginal spending propensity of the whole household sector in the simplified case of having only two income groups. The changes in the total disposable income and consumption of the social classes caused by the government stimulus are shown in Figure 28 (disposable income is almost equally split between two social classes). It can be seen that the spending multiplication effect is mainly produced by the increase in consumption of low-income households.

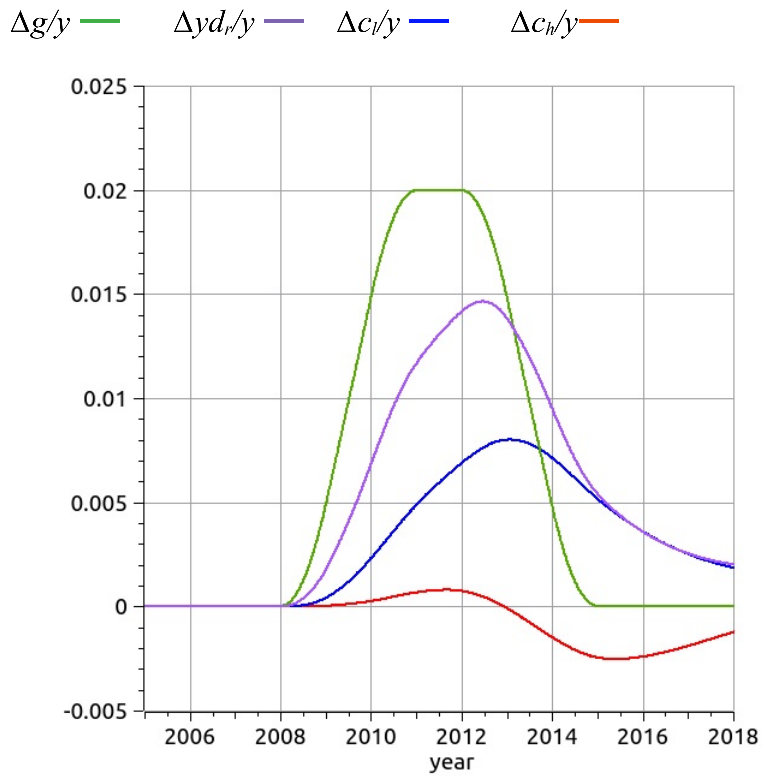

Figure 28: "FiscalStimulus", household disposable income and consumption.

The evolution of real corporate investment is shown in Figure 29. This is the "investment accelerator" component of the "supermultiplier". The increase in corporate investment is caused by capital utilisation rate rising temporarily above its normal value, as shown in Figure 30.

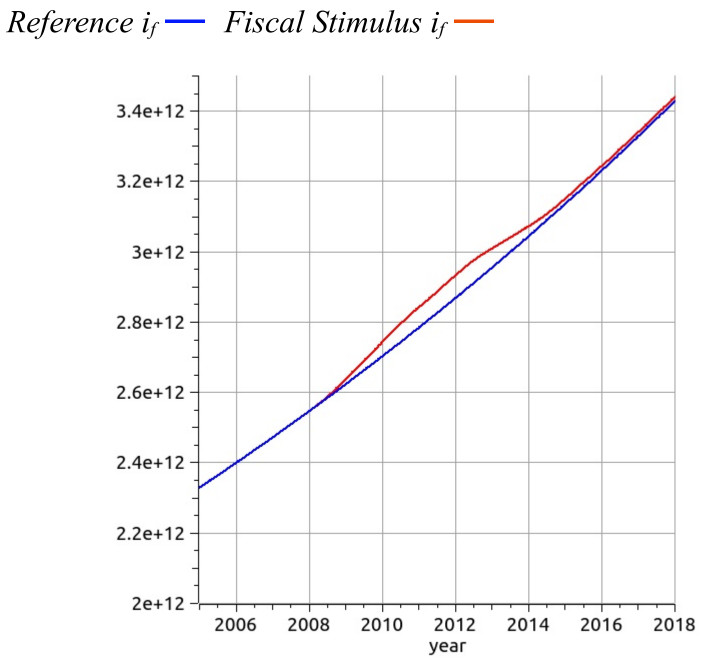

Figure 29: The investment accelerator. Real corporate investment in fiscal stimulus and reference scenarios.

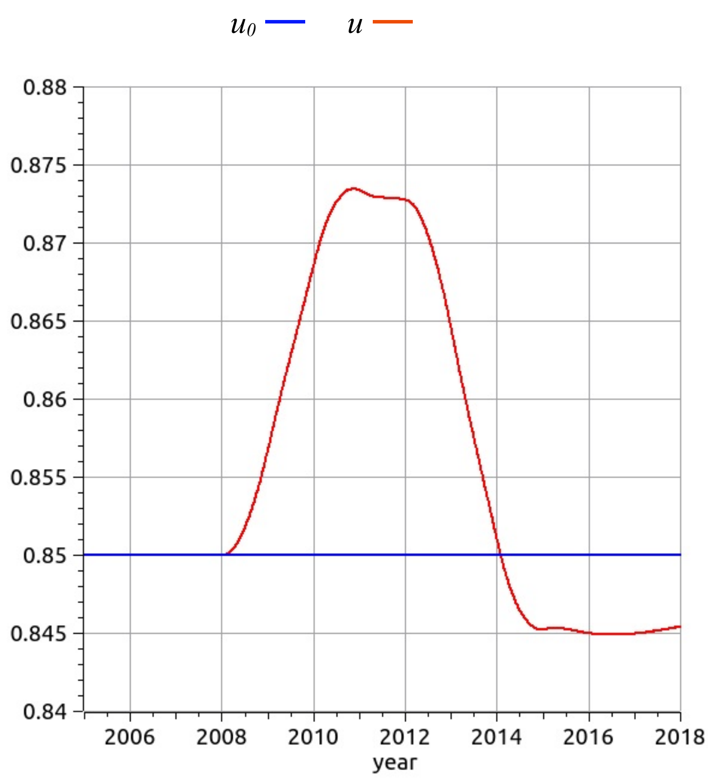

Figure 30: The investment accelerator, capital utilisation.

4.9.2 The sensitivity of the model to changes in the portfolio allocation function and to demand for housing assets

The portfolio allocation function is inspired by the Tobin asset demand system presented by Godley and Lavoie (2007, p.395) but the exogenous parameters have been modified in order to ensure the model stability (preventing the value of the stock of government securities held by the household sector to fall below zero in the historical scenarios). The sensitivity of the model to the changes in the values of the parameters of the portfolio allocation function has been validated by running auxiliary scenarios and checking the behaviour of the relevant parameters. The impact of changes in the following parameters was evaluated: λ_{20}, λ_{22}, λ_{24}, λ_{25}, λ_{40}, λ_{42}, λ_{44}, λ_{45} and η.

Changes in two parameters have been found to have significant impact on the level of GDP: λ_{40} (the fraction of investible wealth of high-income households allocated to equities), and η (the ratio of new mortgage loans to disposable income). The λ_{40} elasticity of GDP is around 0.2, the short-run η elasticity of GDP is around 0.1 and long-run η elasticity of GDP is around 0.2.

Changes in other parameters only affect the allocation of financial assets.

4.10 GDP growth trajectories in the long run

GDP growth trajectories depend on the ratio of overall utilisation of the capacities of the economy (including the labour) and long-run effects on the productivity growth. The graph presented in Figure 31 illustrates exponential growth of the GDP ("Reference" scenario), the trajectory generated when aggregate demand has been reduced (without reducing the productivity growth; "FiscalExperiment" scenario) and the trajectory generated when the productivity growth rate has been reduced ("ProductivityExperiment" scenario). In "FiscalExperiment", the rate of growth of real government expenditure "gr_g" has been temporarily reduced (see Section 4.11.1 for more detail). In "ProductivityExperiment", scenario ER_{pr0} and ER_{pr1} thresholds were changed to force the productivity growth to be lower. The changes in both experiments were limited in scale to ensure that the model was still operating in the linear range.

The growth trajectory affected by reduced aggregate demand (simulated by reduced government expenditure) is, on a graph with a logarithmic scale, parallel to the line describing the “Reference” scenario and shifted downwards. This demonstrates a “level effect”, as explained by Nikiforos (2018). If the productivity growth rate has been reduced, the line has a lower gradient. Both effects can lead to hysteresis. This is self-evident with the productivity losses. A slower rate of accumulation due to lower aggregate demand leads also to a lower stock of wealth affecting consumption in the long run. Small losses caused by the output gap due to insufficient aggregate demand can be reversed but losses in the actual GDP growth due to lower productivity growth are irreversible, as long as foreign technology transfer does not occur, because labour productivity is limited by the available technology. The actual model seems to be asymptotically stable as long as state variables remain within the sensible ranges from the economic point of view. It is possible to modify the model so that it generates an endogenous cycle but demonstrating this falls outside of the scope of this study.

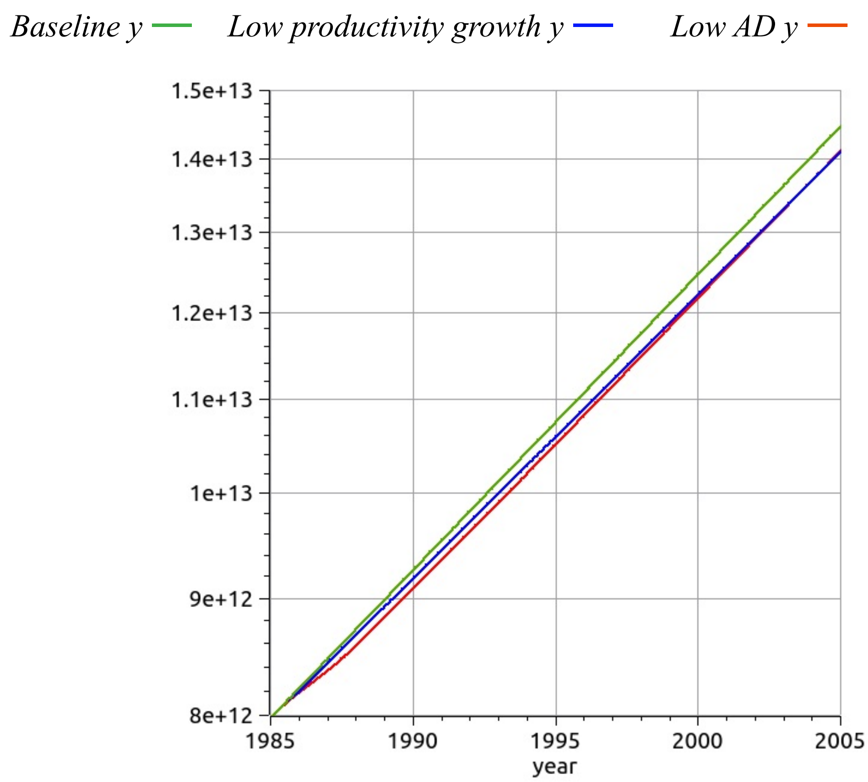

Figure 31: The impact of low productivity growth and insufficient aggregate demand on the long-run GDP growth trajectory

4.11 Simulation of responses of the model to changes in individual parameters

The scenarios presented in this section involve changing individual parameters of the already calibrated steady-state growth model, while dynamic responses of the model remains linear and there are no productivity losses causing hysteresis in the long run. Similarly, in Section 4.12, more complex scenarios are simulated but the shocks are also small to ensure that the effects are linear. The scenarios presented in Sections 4.11 and 4.12 are not calibrated to reproduce the changes which occurred in the actual economy. An attempt to reproduce the actual historical trajectories is presented in Section 4.13.

4.11.1 Permanent reduction in government spending

The response of the model to a short-time fiscal stimulus has already been presented in Section 4.9. A small reduction in government spending (Figure 32) can be simulated by temporally reducing the exogenous rate of growth of government expenditure "gr_g", while the parameter "ε_{GYR}" has been set to 0 (scenario "FiscalExperiment"). There are 2 ways of controlling government expenditure in the model, either by directly modifying the rate of growth "gr_g" or by changing the long-run target ratio "GYR" while the adjustment parameter "ε_{GYR}" is greater than 0. Since the reduction is small and the system is linear, the short term response only involves the (super) multiplier which amplifies the reduction in government spending by the factor of 1.6 (Figure 33). The long-term effects of fiscal withdrawal are related to capital accumulation and possibly changes in the rate of productivity growth. The negative wealth effects lead to gradual reduction of consumption spending of high-income households, as shown in Figure 34. The long-term changes in consumption are much larger in magnitude than the short time impact of the supermultiplier. This effect is similar to what has been described in Chapter 3.4 of Godley and Lavoie (2007). The long-term government spending multiplier is greater than 2.5. This result might not be entirely correct and is further discussed in Section 4.16.3.

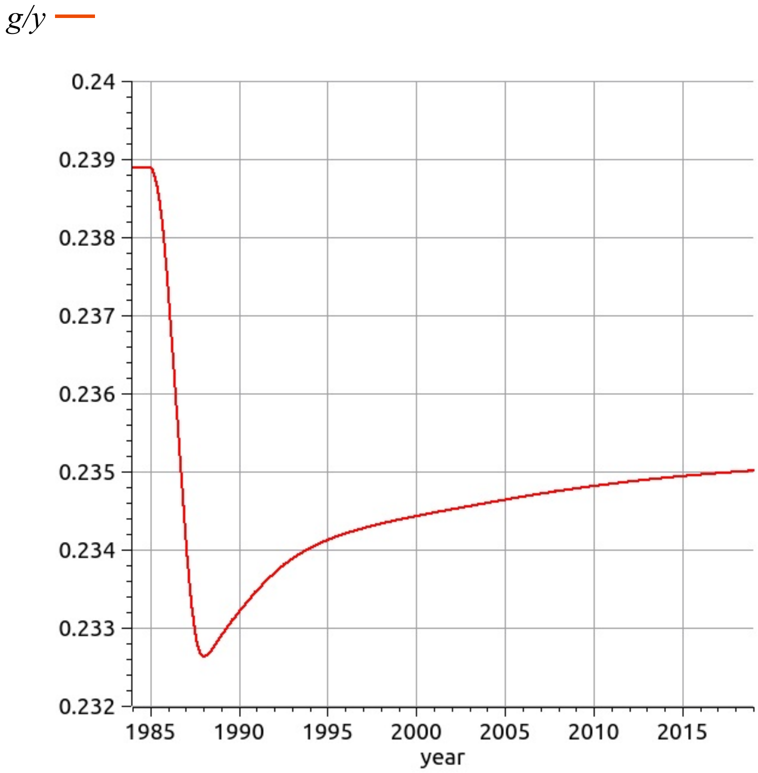

Figure 32: "FiscalExperiment" scenario, rate of government expenditures to GDP.

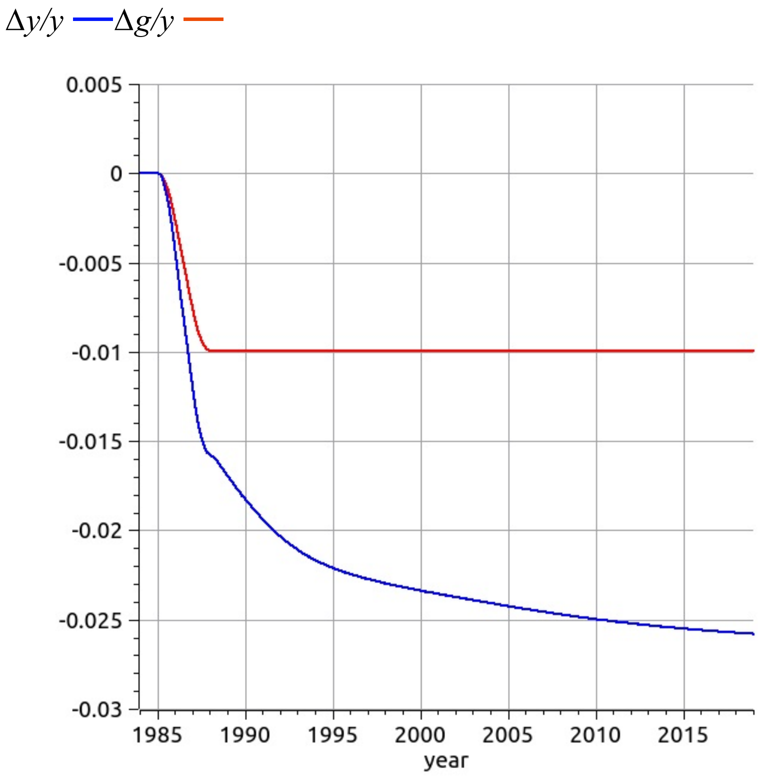

Figure 33: "FiscalExperiment" scenario, spending multiplier in the long run.

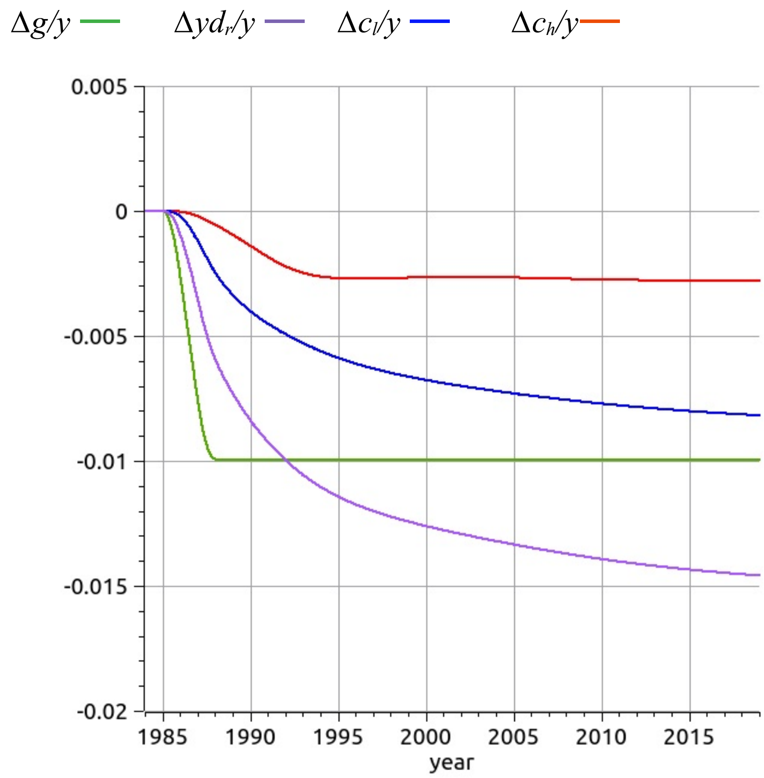

Figure 34: "FiscalExperiment", household disposable income and consumption.

4.11.2 Distributional changes

This is one of the key simulations performed with the model, demonstrating the impact of distributional changes on the GDP trajectory (Figure 36). Unlike in traditional Marxist and Post-Keynesian models where only workers earn wages and capitalists receive profits, disposable income distribution between low-income and high-income classes (Figure 35 and Figure 37) depends not only on the rate of profit but also on the ratio between the base wage paid for simple labour and the salaries paid to professionals and managers for what is considered to be complex labour. In "DistributionalChanges" scenario the fraction of the total demand for labour which is satisfied by high-income households "HILD" has been gradually increased from 0.4 to 0.41, to generate a small shock which does not lead to excessive unemployment causing deflation and productivity losses. The actual changes which happened in the American economy were more significant as documented by Elsby et al. (2013) but they were partially offset by other processes (and masked by the dot-com and housing booms). An attempt to simulate distributional changes of the magnitude which was observed in the American economy without the presence of other processes leads to model instability. Another dimension of the distributional changes in the American economy was the increase in the share of profits in the GDP and the relative reduction of the labour income (mentioned in Section 4.1.4). This process will be included in historical scenario simulations in Section 4.13, together with a larger-scale change in the value of "HILD". An interesting side effect of the distributional changes appearing in the model is an increase in budget deficits (Figure 38) and long-term growth of the public debt to GDP ratio (Figure 39). This can be explained by an increased share of high-income households disposable income in the total disposable income. It is assumed that low-income households consume all their income while high-income households consume a fixed fraction of their expected real wealth in a unit of time. Since aggregate demand is always equal to aggregate supply, the wealth to GDP ratio has to increase due to the reduction of the marginal spending propensity of the whole household sector. This is shown on Figure 20 as the changes in the gradient of the line depicting the AD (aggregate demand) schedule. An increase of the relative wealth of the high-income households is achieved by saving more financial assets, mainly government securities and money, while the GDP is lower than in the reference scenario.

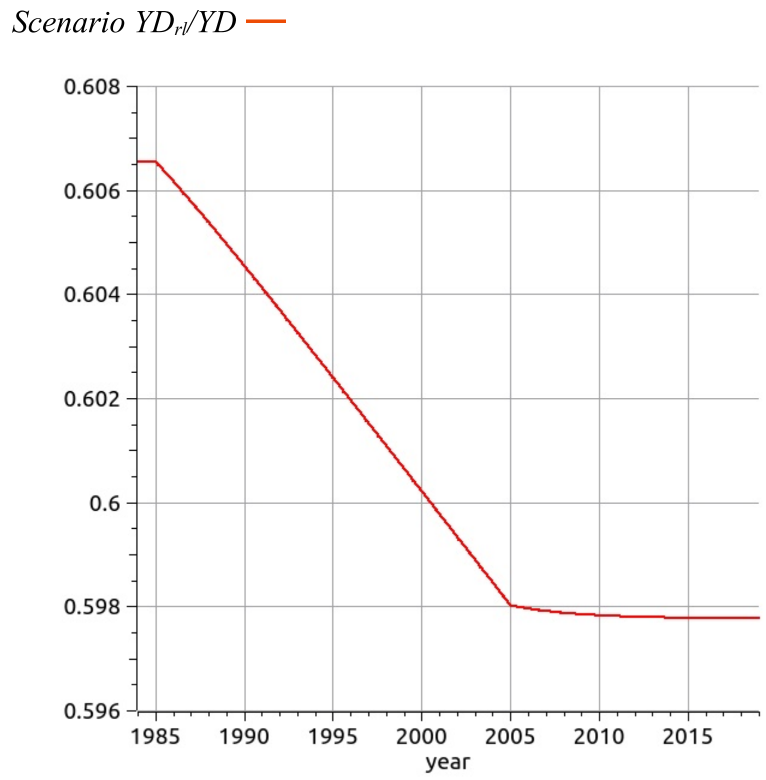

Figure 35: “DistributionalChanges” scenario, disposable income distribution.

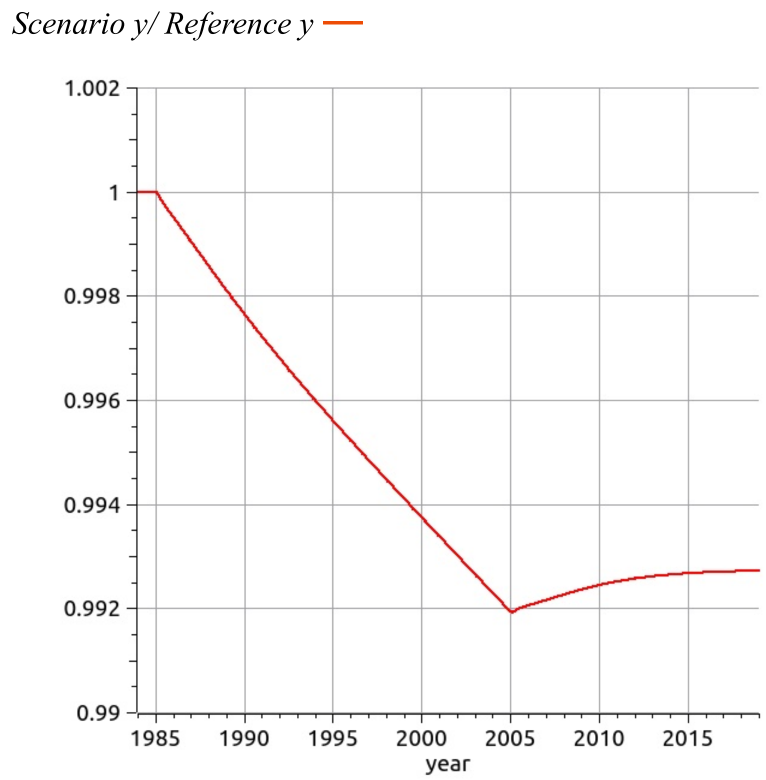

Figure 36: “DistributionalChanges” scenario, impact on GDP.

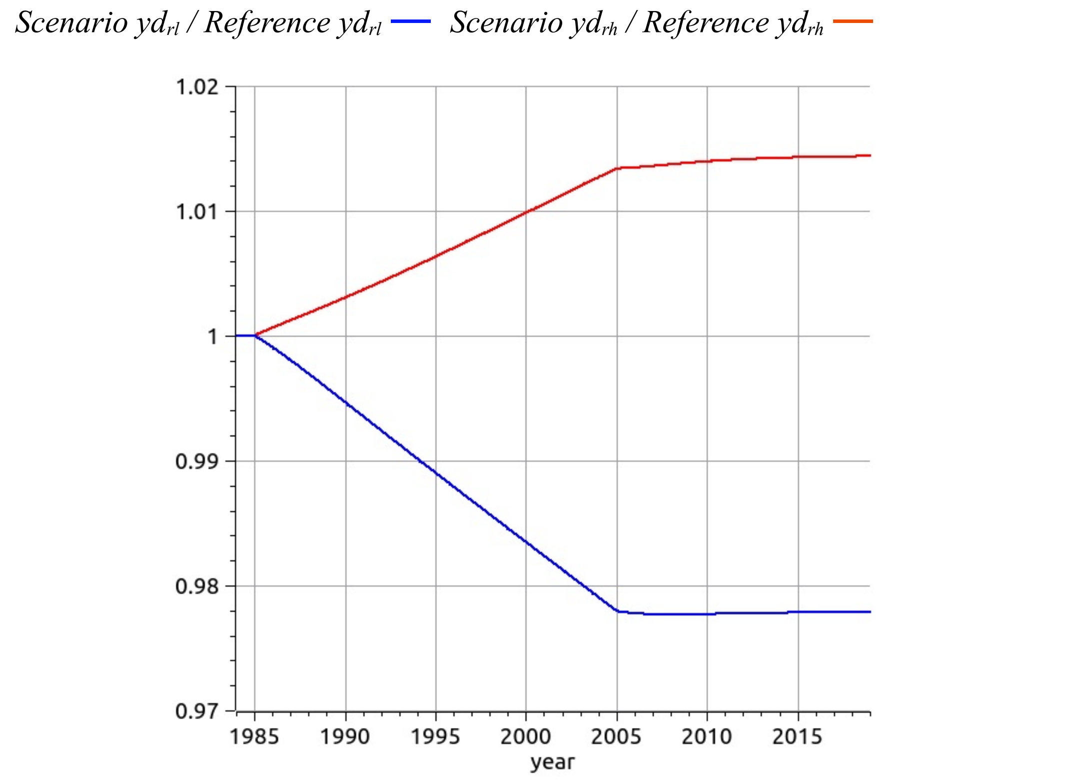

Figure 37: "DistributionalChanges" scenario, impact on disposable income.

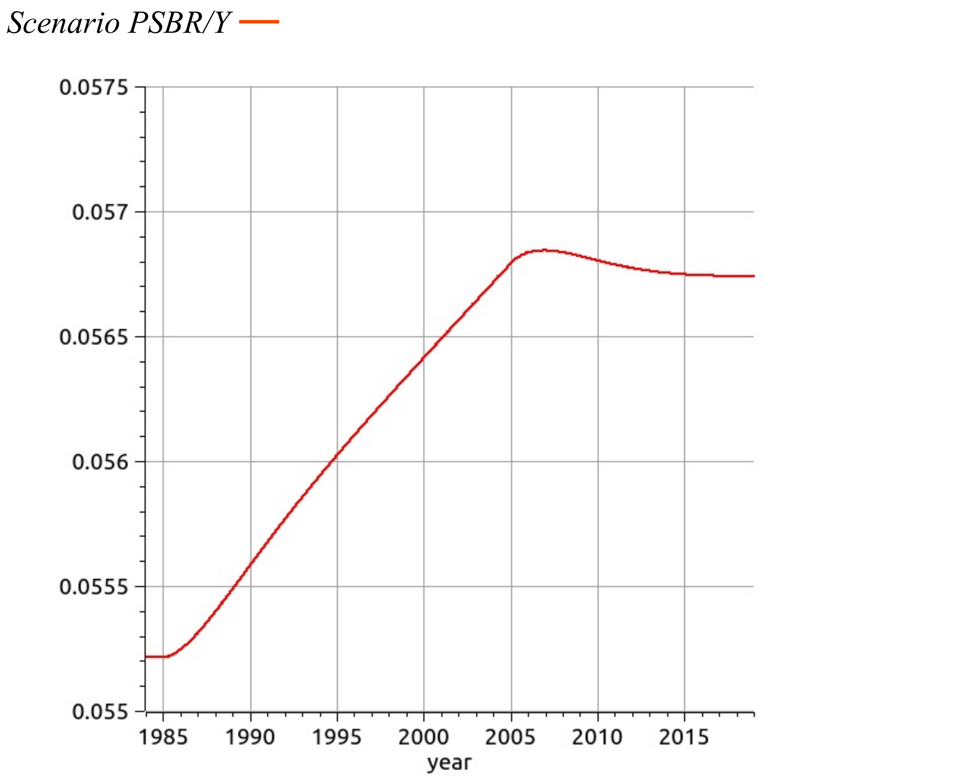

Figure 38: "DistributionalChanges" scenario, impact on budget deficit.

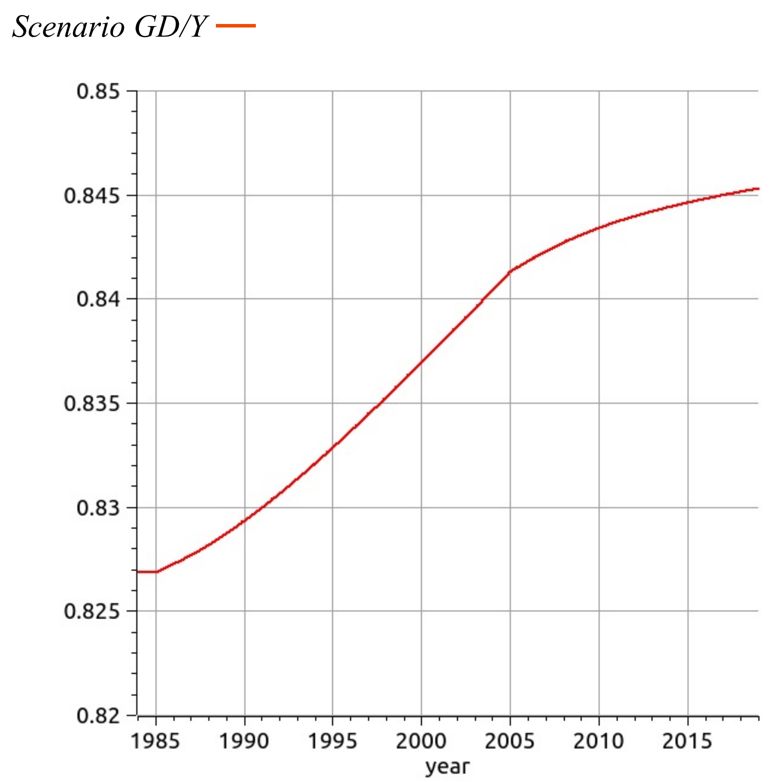

Figure 39: "DistributionalChanges" scenario, impact on public debt to GDP ratio.

4.11.3 Changes in monetary policy

Changes in monetary policy in the "MonetaryStimulus" scenario are simulated by lowering the deposit interest rate "r_m" by 100 base points (from 4% to 3%) between 1991 and 1992 and leaving it at 3% until the end of the simulation period. In the model, a monetary stimulus works mainly by redistributing income towards low-income households (Figure 41), not by stimulating investment. There are also some wealth effects caused by the changes of portfolio allocation (Figure 42). As argued by Sharpe and Suárez (2015), unless the lending interest rate exceeds a threshold called a "hurdle rate", corporate investment is not affected strongly by changes in monetary policy. There is also no explicit dependency of private residential investment on the interest rate in the model as it is difficult to find the relevant econometric data. Comparing with the baseline model there is a moderate positive medium-time impact of a monetary stimulus (about 0.5% increase in real GDP for a 1% reduction in the deposit interest rate - Figure 40). If investment was made significantly dependent on the changes of interest rates the model would become oversensitive to changes in monetary policy and generating realistic-looking trajectories would become impossible, which confirms the Post Keynesian position presented and discussed in Section 2.4.

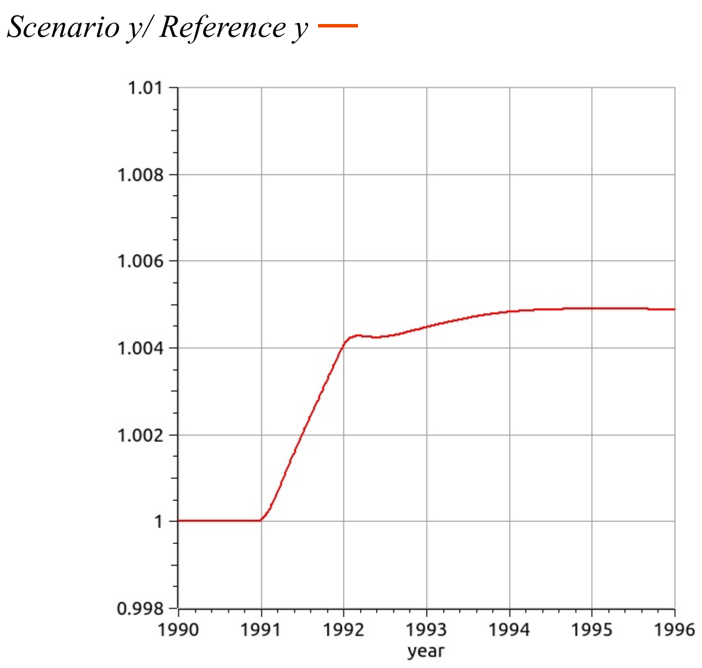

Figure 40: "MonetaryStimulus" scenario, impact on GDP.

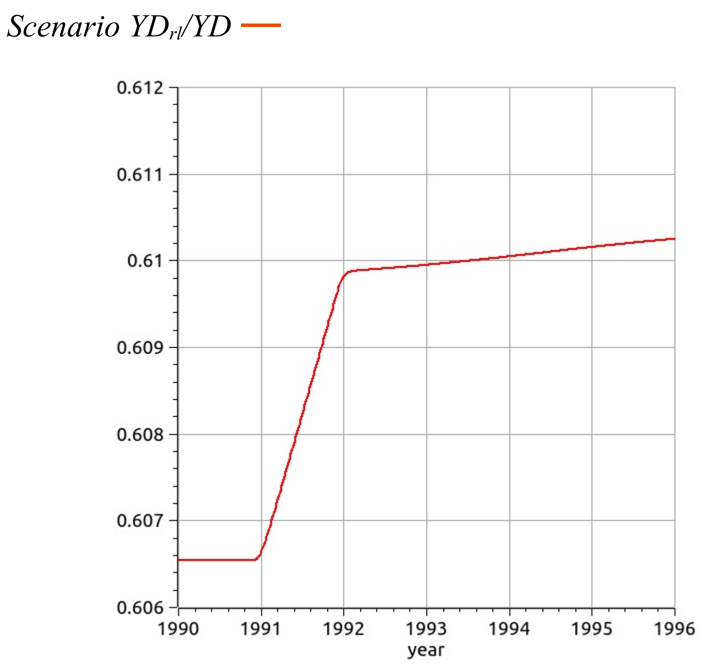

Figure 41: Impact of the interest rate reduction on disposable income distribution.

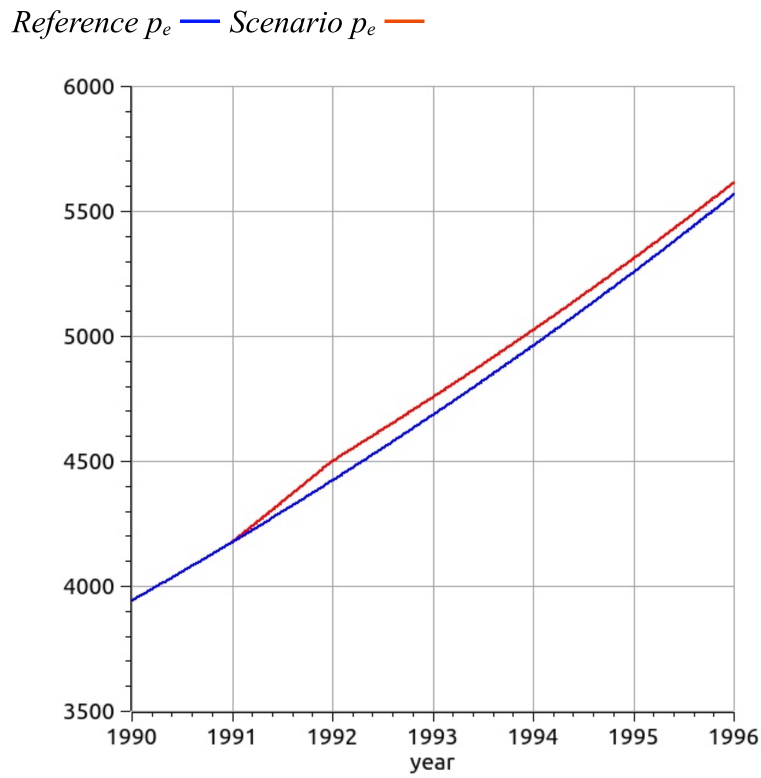

Figure 42: Impact of the interest rate reduction on the price of equities

4.11.4 Trade balance changes

Changes in the trade balance affect the GDP in a similar (but not identical) way as changes in autonomous government expenditure ("TradeBalanceChanges" scenario) – see Figure 43. A multiplier effect is also present. The parameter which is changed in the simulation is average net import propensity, "μ".

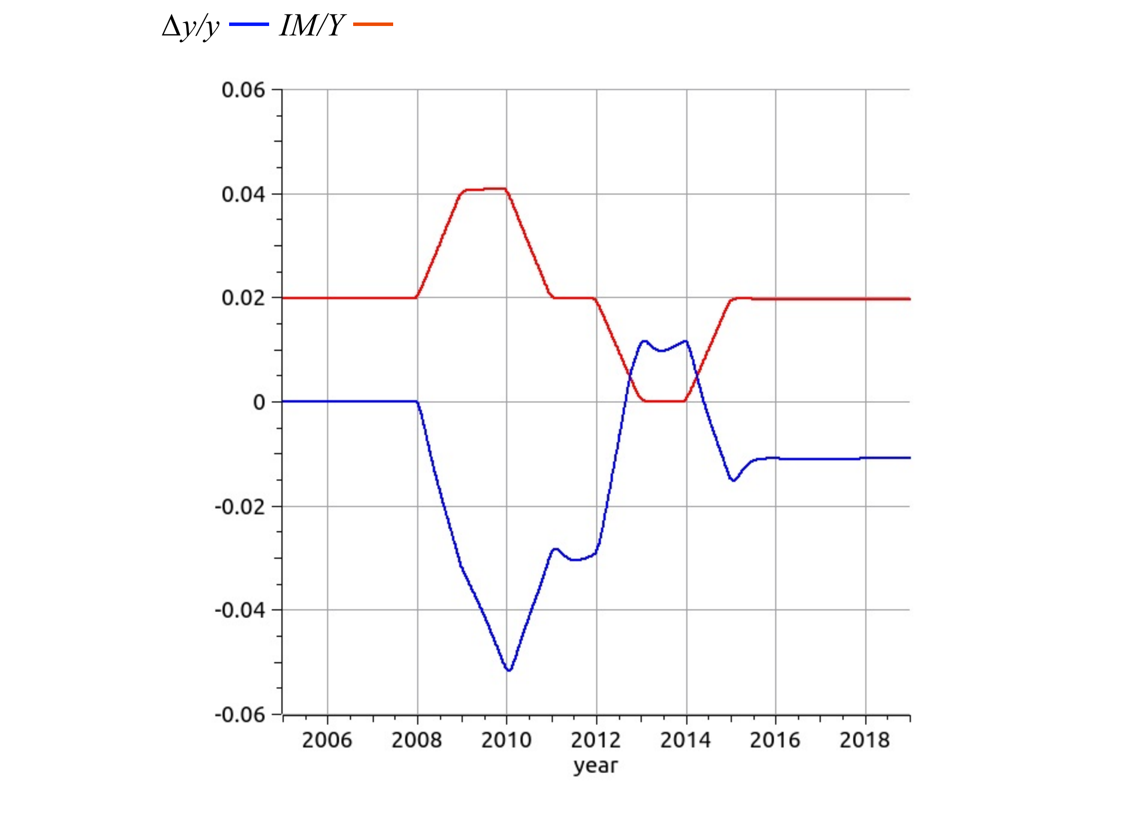

Figure 43: "TradeBalanceChanges" scenario, impact of trade balance on GDP

4.12 Simulation of complex dynamic scenarios

4.12.1 A stock market bubble

In this scenario both the equities portfolio choice parameter "λ_{40}" and target capital utilisation ratio "u_0" are changed to simulate a temporary increase in the price of equities and a temporary increase in the willingness to invest in productive capital. These processes unfolded during the dot-com bubble with an even greater magnitude than in this simulation. Changes in exogenous parameters are shown in Figure 44 and Figure 45. The impact on Tobin's q ratio is illustrated in Figure 47. The magnitude of the simulated bubble is large enough to push the unemployment rate, shown in Figure 49 above the threshold triggering rising wage expectations (the model's response may not be fully accurate due to the low Okun's coefficient). Lower unemployment leads to a temporary spike in inflation, shown in Figure 50. When the bubble bursts, investment (shown in Figure 46) is subdued which causes a shallow slump (what can be seen in Figure 48). The scenario is simplified as corporate defaults are not simulated.

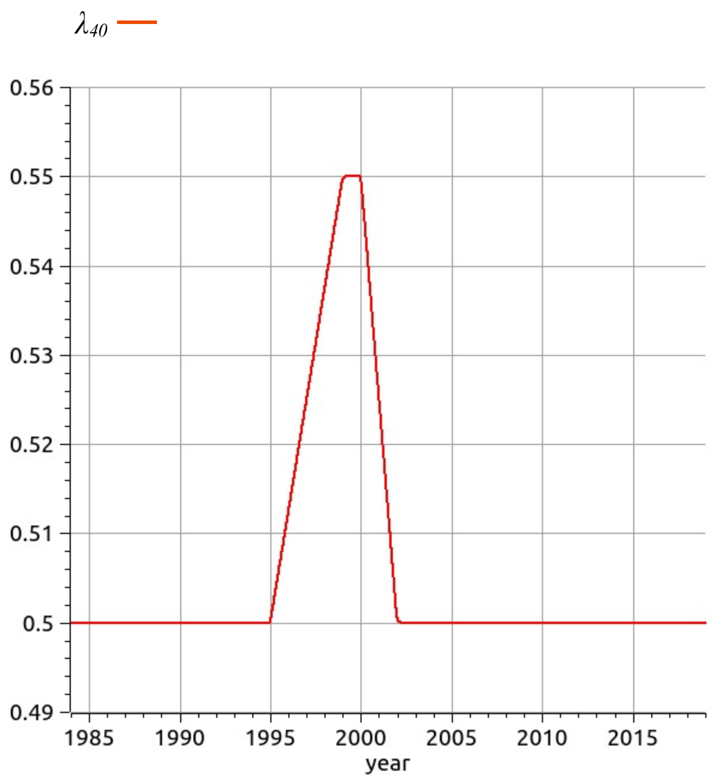

Figure 44: "StockmarketBubble" scenario, equities portfolio choice parameter.

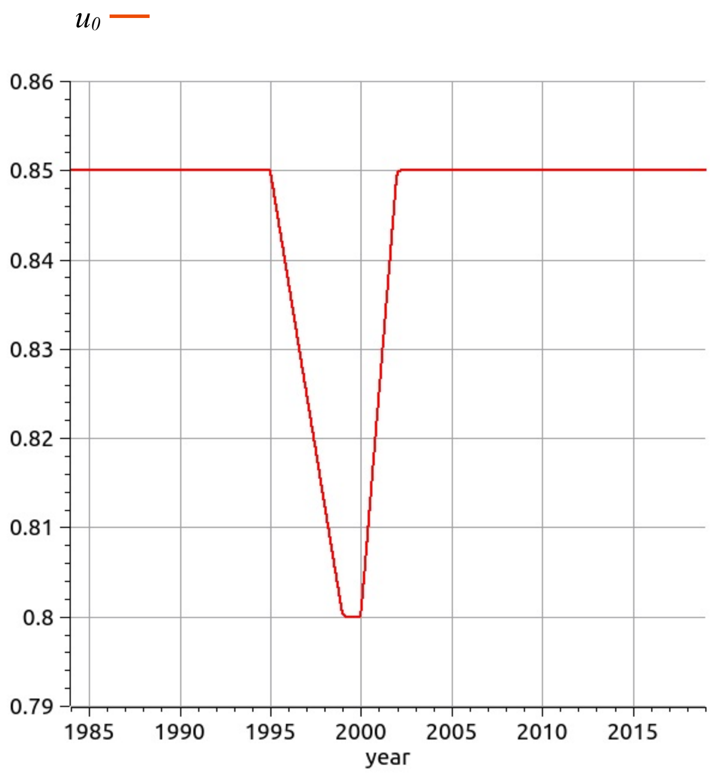

Figure 45: "StockmarketBubble" scenario, target capital utilisation rate.

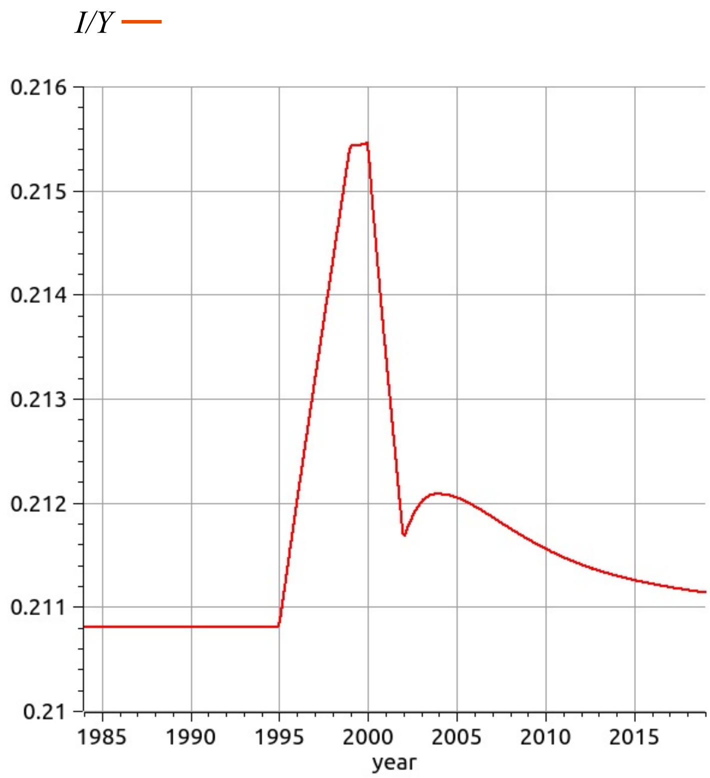

Figure 46: "StockmarketBubble" scenario, impact on private investment to GDP ratio.

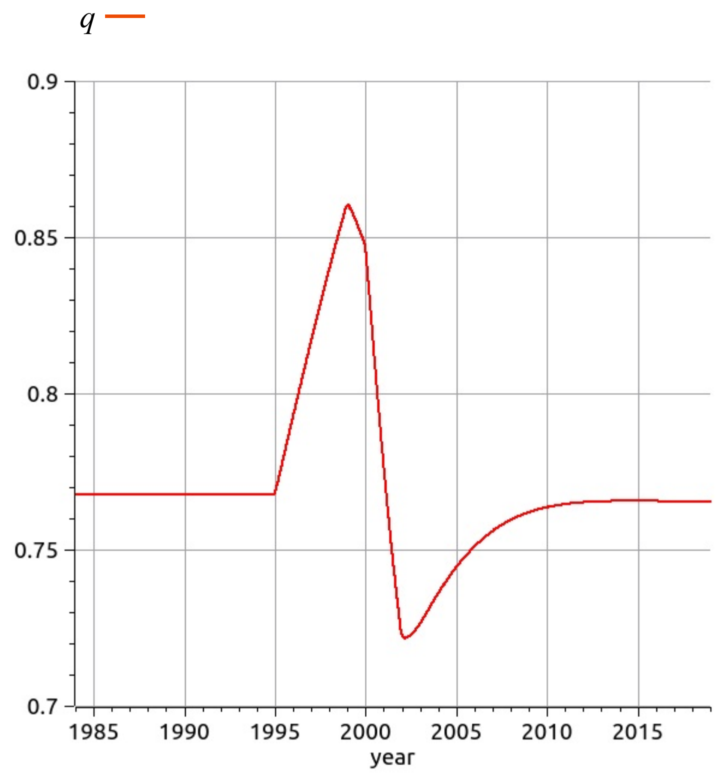

Figure 47: "StockmarketBubble" scenario, impact on Tobin's q ratio.

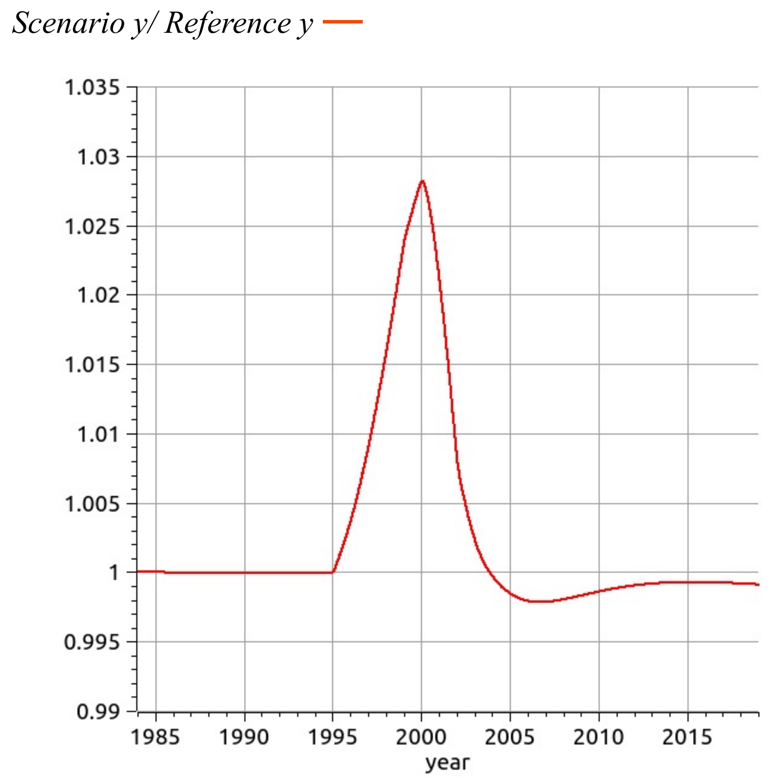

Figure 48: "StockmarketBubble" scenario, impact on GDP.

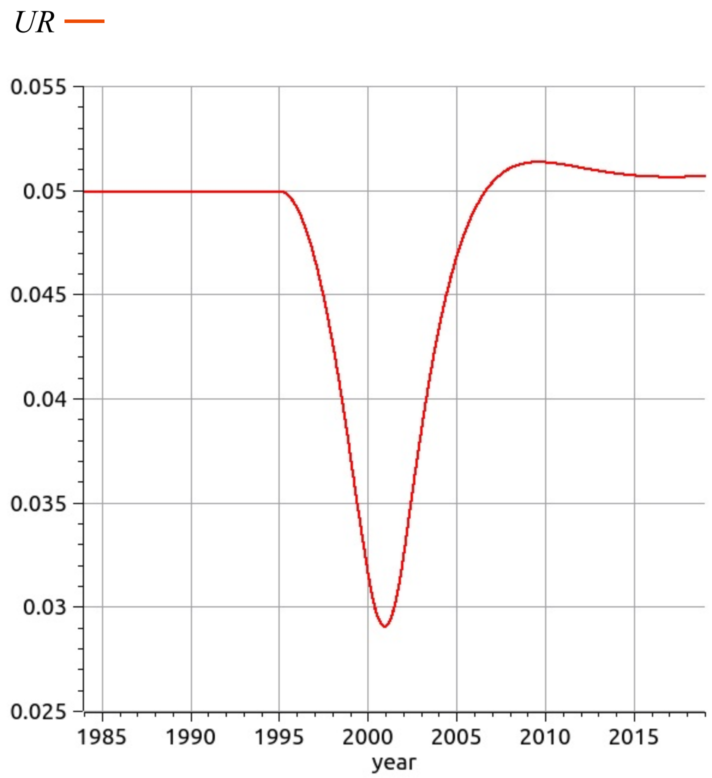

Figure 49: “StockmarketBubble” scenario, impact on unemployment rate.

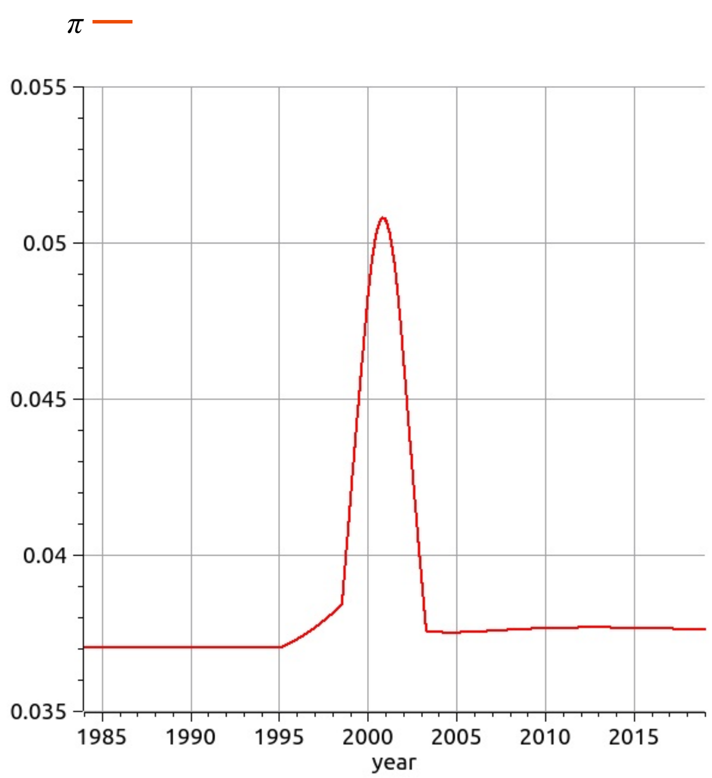

Figure 50: StockmarketBubble scenario, impact on inflation rate.

4.12.2 A housing market bubble and crash

It is possible to simulate a bubble by endogenously changing "η", the parameter which determines the ratio of gross mortgage lending to total disposable income (Figure 51). In the model the value of real estate assets depends on the stock of mortgages. Two scenarios have been implemented to further examine the impact of wealth effects on the aggregate demand. Constant rate of productivity growth is assumed. In the "HousingBubble" scenario the ratio of the value of real estate assets to the stock of mortgages "REMOR" remains constant; in "HousingBubblePriceCrash" scenario a slump in mortgage lending is accompanied by a temporary reduction in the valuation of real estate assets (Figure 52). The value of real estate assets consists of the value of land and housing structures (Figure 54). The total value of land is the component which is linked with the stock of mortgages while the value of housing structures is determined by the cost of building them. Housing structures depreciate at a fixed rate. The evolution of investment to GDP ratios is shown in Figure 55 and Figure 56, the changes in wealth components are presented in Figure 57 and Figure 58. The evolution of the disposable income and household consumption is shown in Figure 59 and Figure 60.

What stimulates the economy in the initial phase of the cycle, the housing bubble, is an increase in mortgage borrowing, increasing the investment and flowing to these households who owned land. The slump is deeper in the "price crash" scenario (GDP is presented in Figure 61 and unemployment rate in Figure 62), which demonstrates the magnitude of wealth effects. The high income households consumption is determined by their expected wealth and it is this component of aggregate demand which collapses the most in the "price crash" scenario, as seen in Figure 60. Since the "HousingBubble" scenarios simulate a shallow boom and bust cycle to avoid the non-linearity in the response of the rest of the economy, the volume of net mortgage lending shown in Figure 53 is never negative so there is no "debt-deflation" proper. The "Historical" scenarios (described in Section 4.13) simulate the actual bubble and debt-deflation of the early 2000s, which had a greater magnitude.

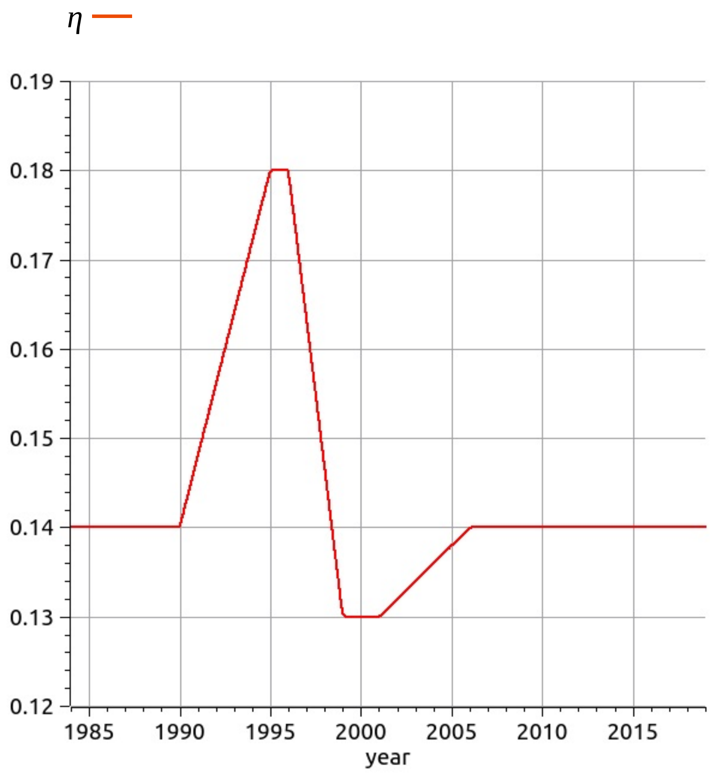

Figure 51: Housing Bubble scenarios, new lending parameter.

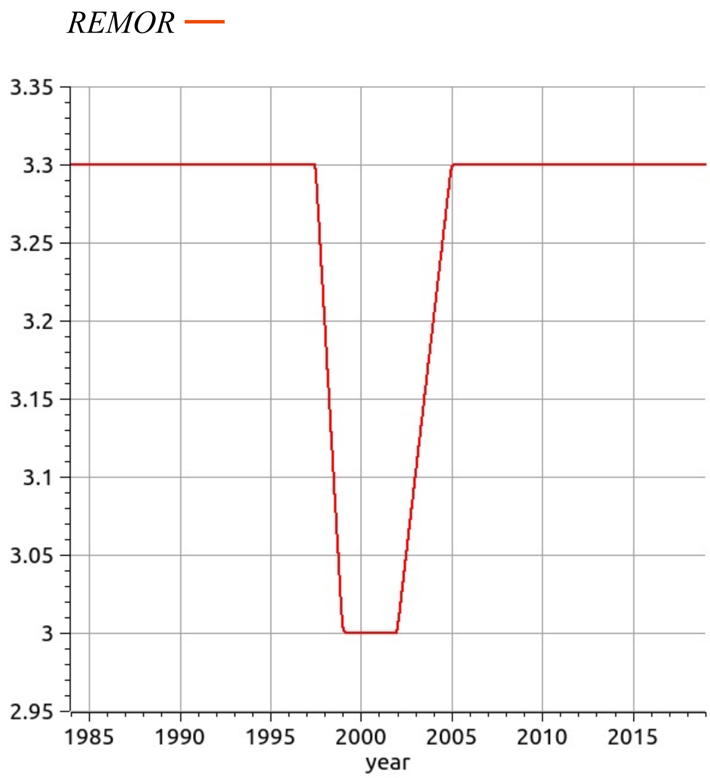

Figure 52: "HousingBubblePriceCrash" house value parameter.

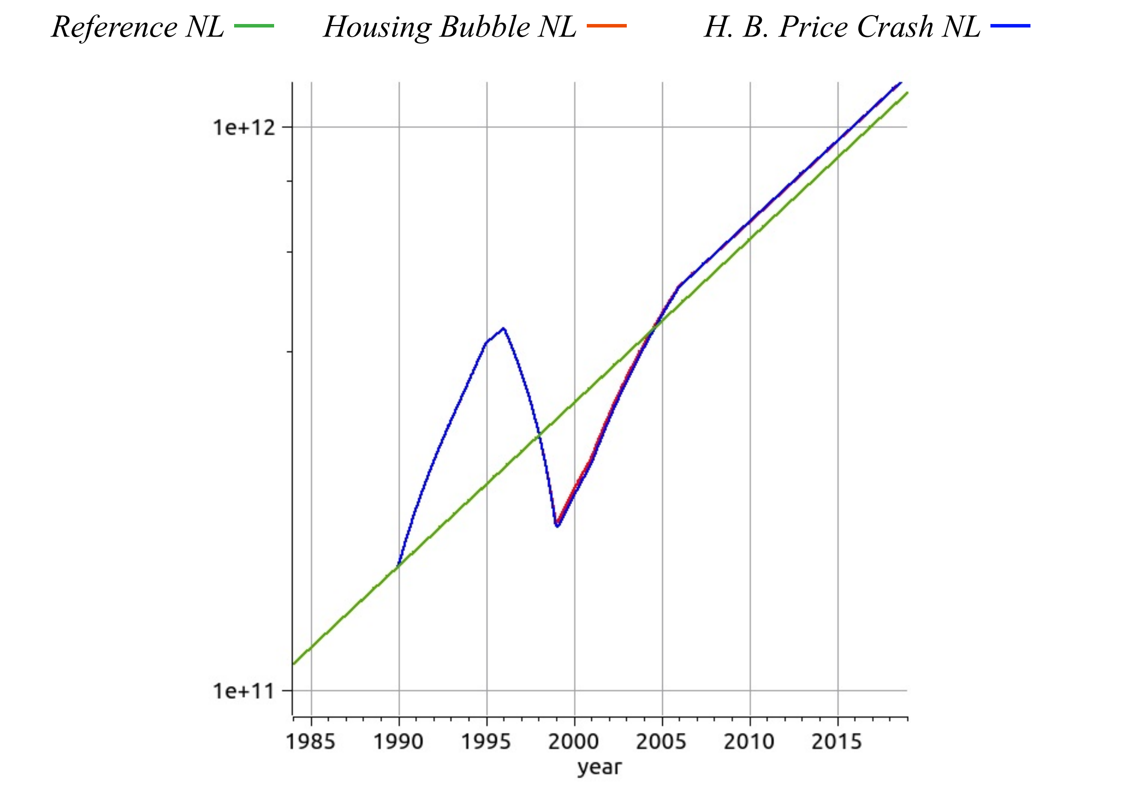

Figure 53: Housing Bubble scenarios, net mortgage lending.

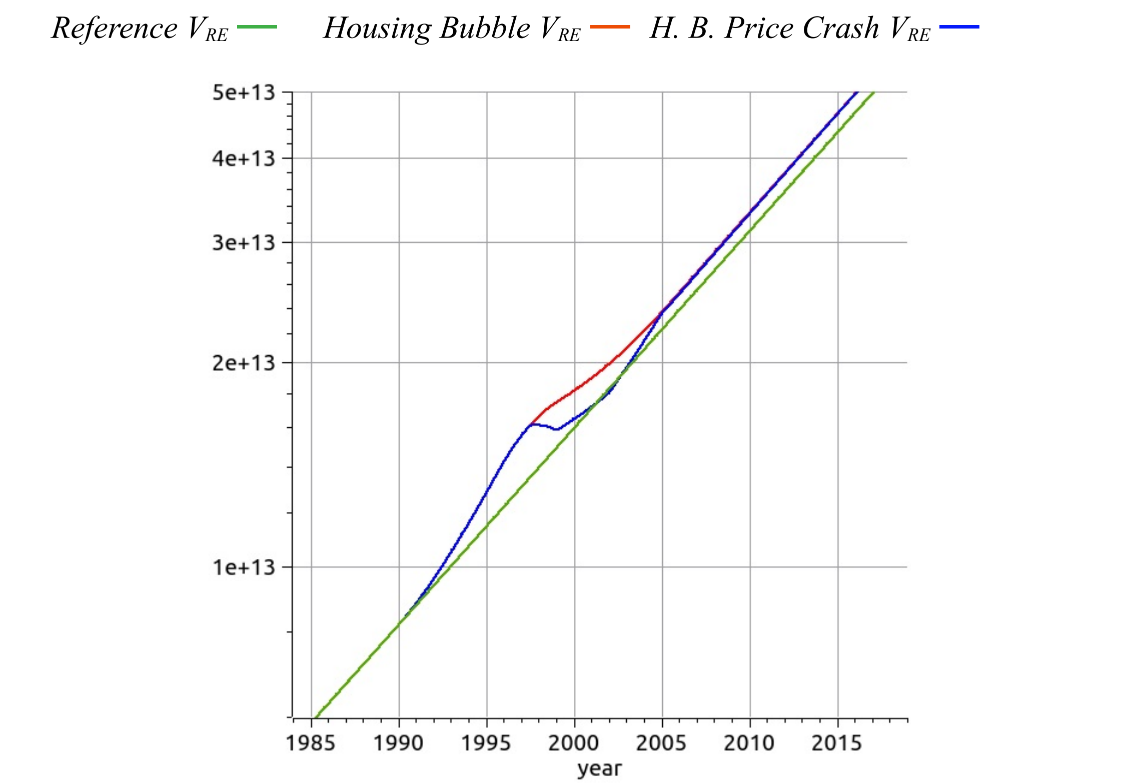

Figure 54: Housing Bubble scenarios, total real estate value.

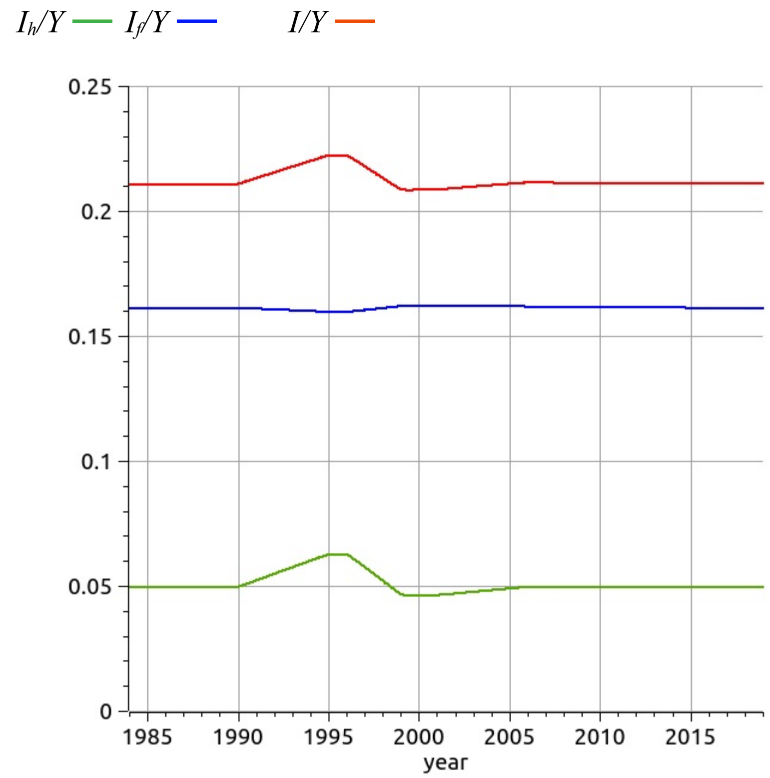

Figure 55: "HousingBubble" scenario, investment to GDP ratios.

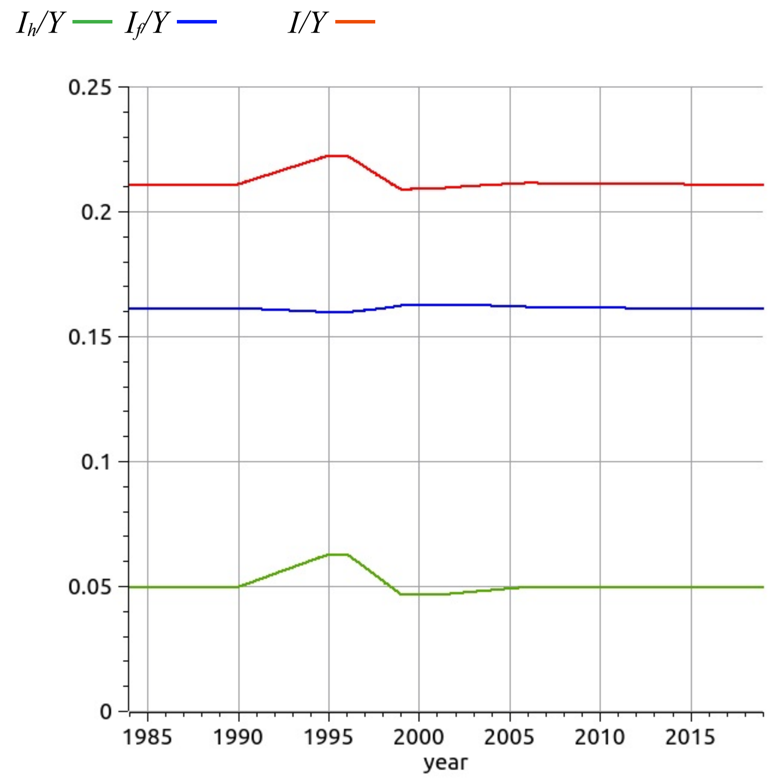

Figure 56: "HousingBubblePriceCrash" scenario, investment to GDP ratios.

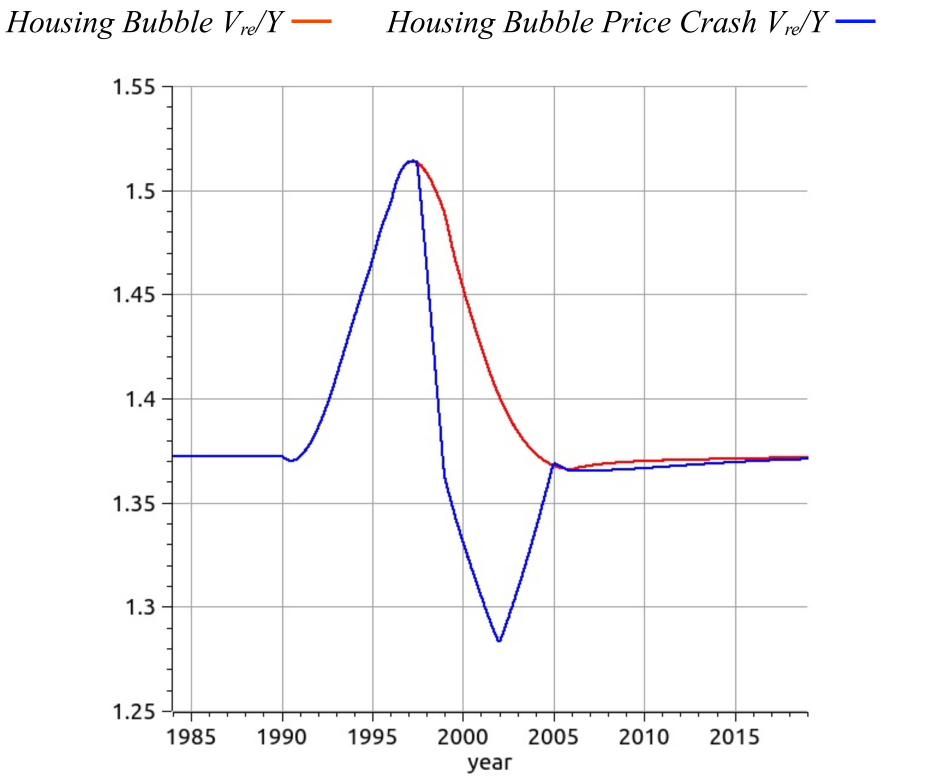

Figure 57: Housing Bubble scenarios, real estate wealth to GDP ratios.

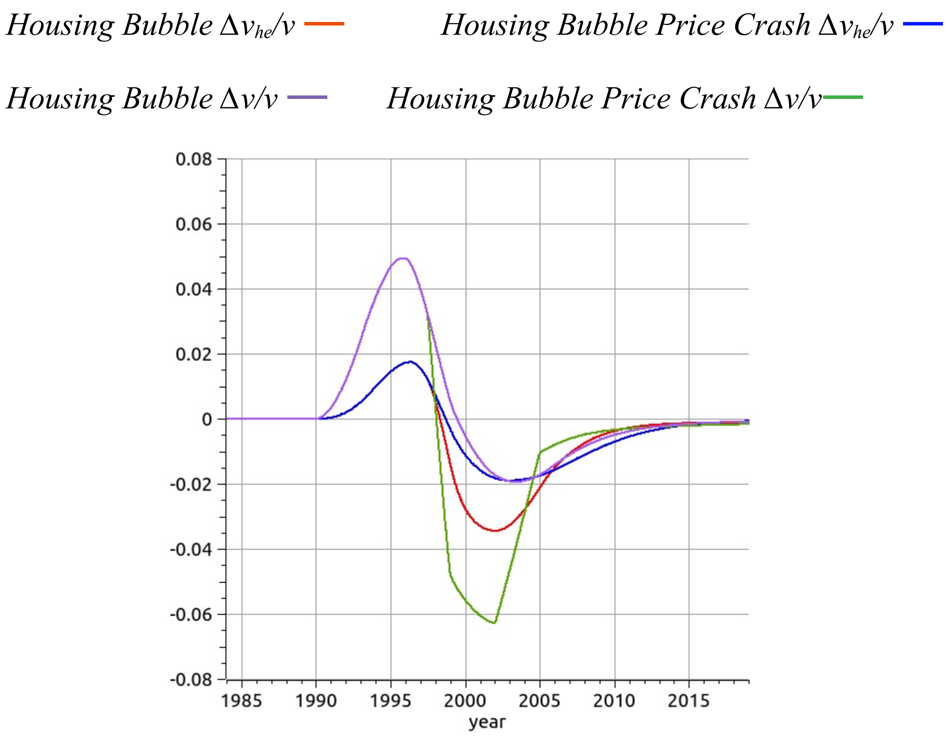

Figure 58: Housing Bubble scenarios, impact on total wealth.

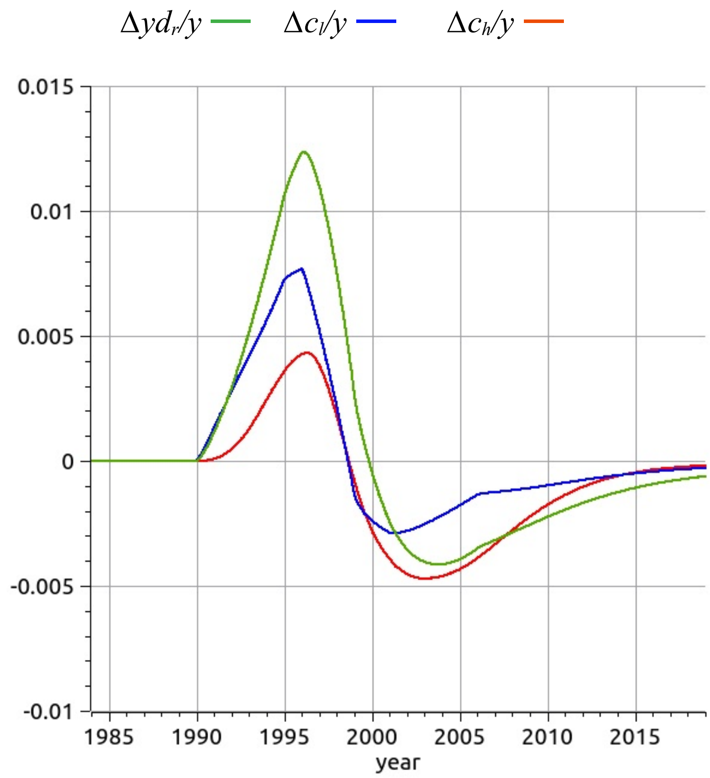

Figure 59: "HousingBubble", household disposable income and consumption.

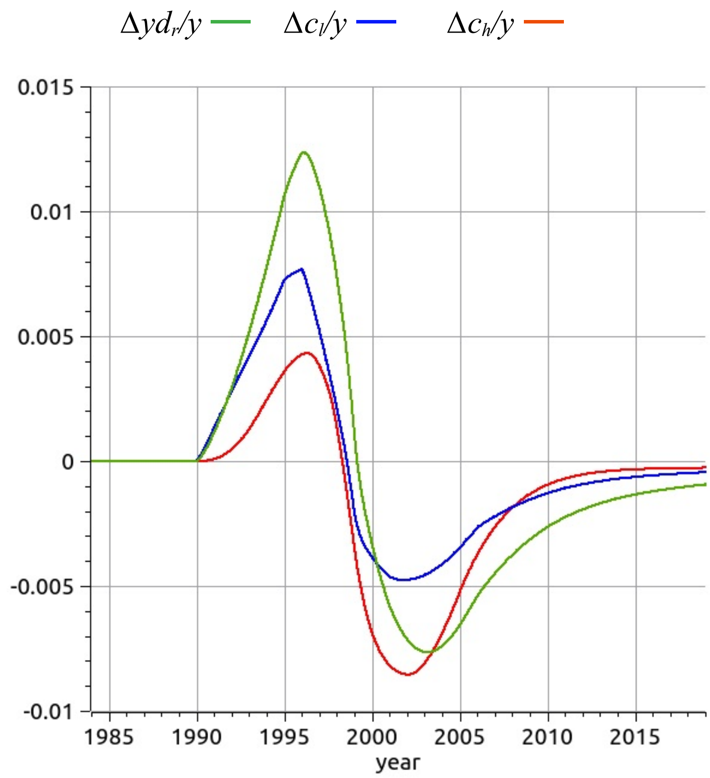

Figure 60: "HousingBubblePriceCrash" household disp. income and consumption.

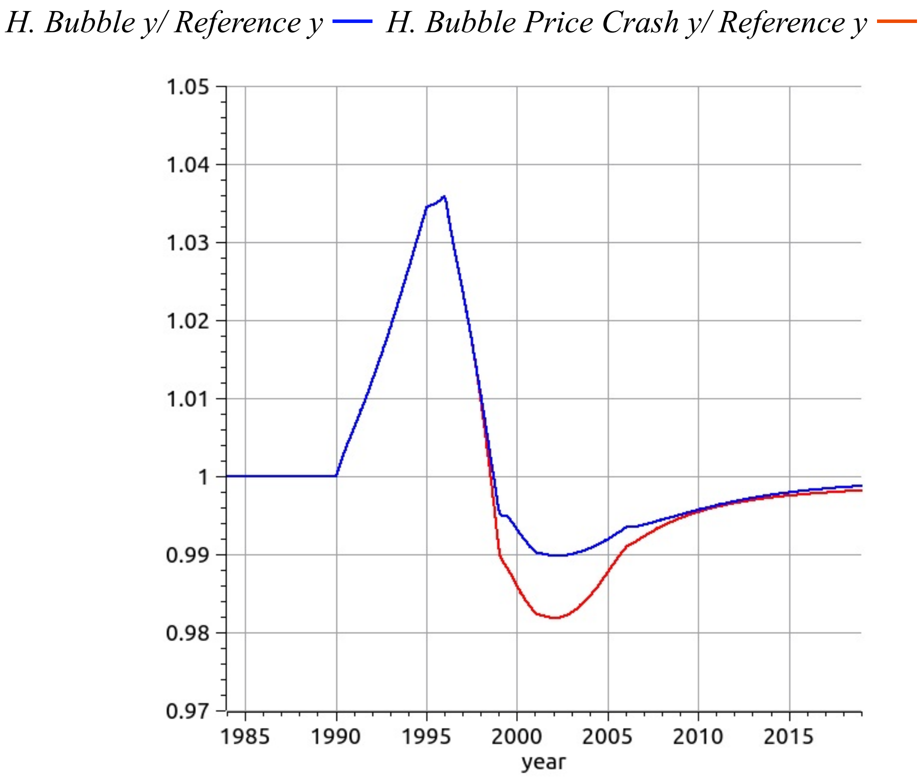

Figure 61: Housing Bubble scenarios, impact on GDP.

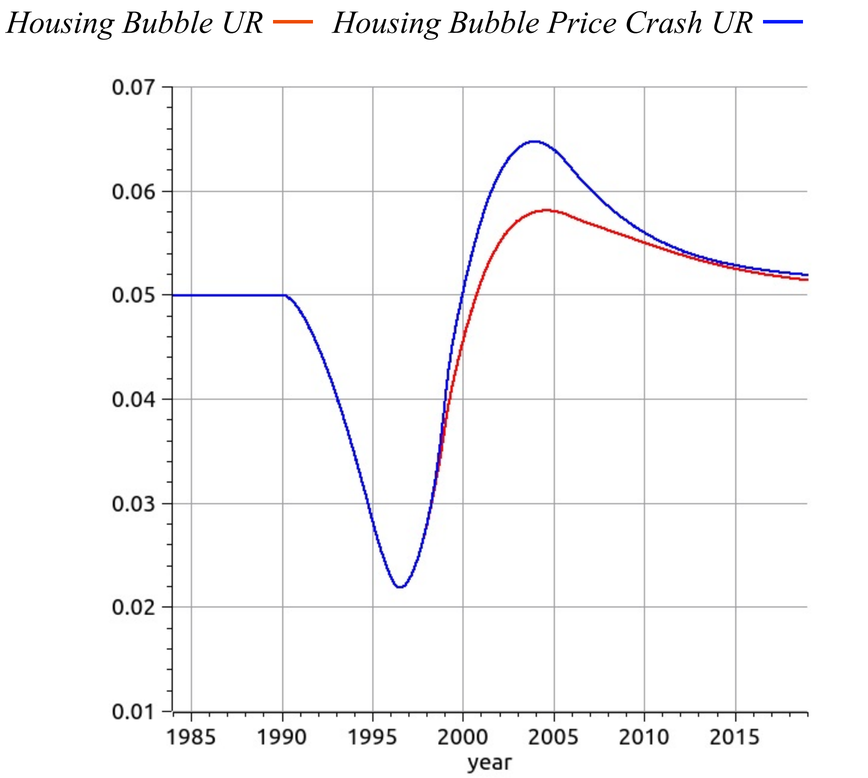

Figure 62: Housing Bubble scenarios, impact on unemployment rate.

Since both Housing Bubble scenarios lead to a negative aggregate demand shock during the crash phase, they may be used to investigate the mechanism of business investment recovery after the crisis. In the model, the rate of corporate investment depends on the difference between the current rate of capital utilisation and the normal rate of capital utilisation. During the boom phase higher demand and high gross domestic product increases the rate of capital utilisation. When aggregate demand falls during the crash phase of the housing cycle, capital utilisation also falls, as illustrated in Figure 63. These changes in the rate of capital utilisation are reflected in the changes of corporate investment (Figure 64). Capital utilisation rate is also reduced over time by capital depreciation. In the absence of further demand and productive shocks (when aggregate demand grows close to the normal rate determined by the rates of productivity and population growth), a negative feedback loop defined in equations (15), (16), (17) and (20) eventually moves back the current rate of capital utilisation "u" close to the normal value "u_0".

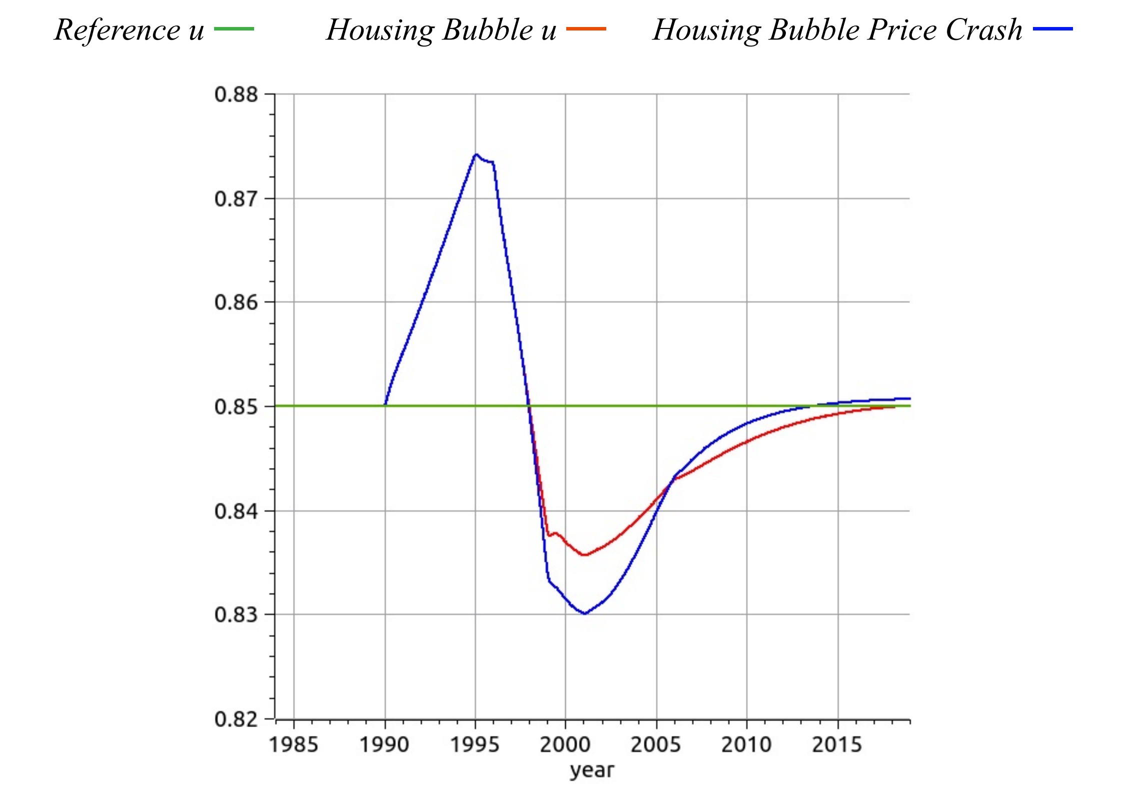

Figure 63: Housing Bubble scenarios, the rate of capital utilisation.

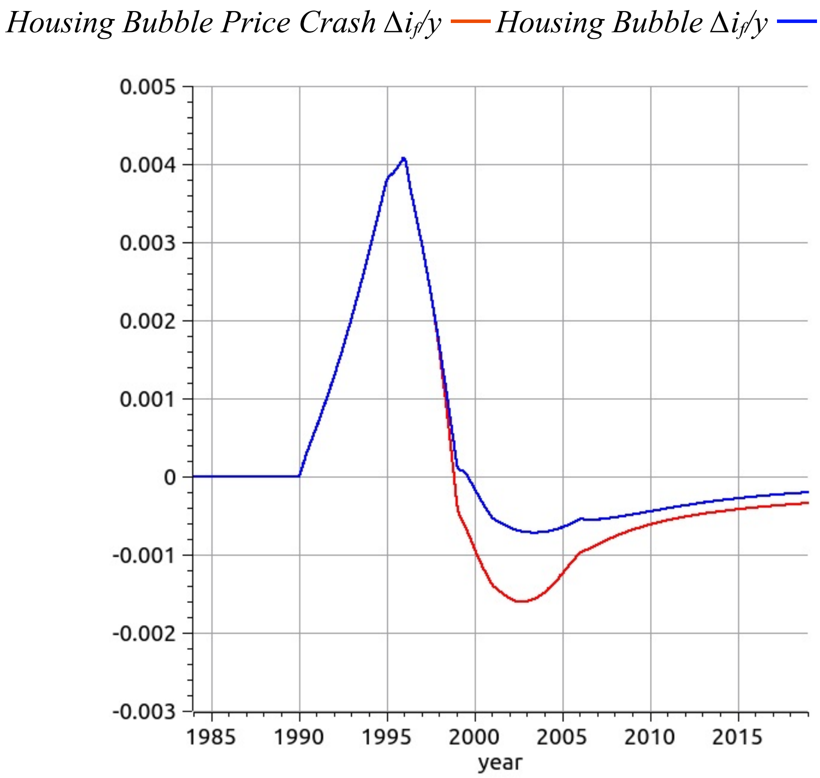

Figure 64: Housing Bubble scenarios, impact on corporate investment rate.

The model has been defined as asymptotically stable so it converges to the normal rate of GDP growth as long as there are no productivity losses. There is a negative (stabilising) feedback loop in the model, involving high-income households wealth accumulation, as illustrated in Figure 57 and Figure 58. It is possible to modify the model so that it generates an endogenous cycle but in this research it has been assumed that the source of instability is exogenous.

4.13 Simulation of historical scenarios

Reproducing the historical scenario has been the main goal of building the model. The following exogenous and endogenous processes have been identified (see Sections 2.5, 2.6, 2.7, 2.8 and 4.1.4):

- A change in income distribution towards high income households.
- Long-term changes in the government expenditure to GDP ratio (caused mainly by the reduction of defence related expenditures after the end of the cold war).
- The dot-com bubble (a stock market bubble).
- The real estate bubble and the GFC.
- The reduction of the rate of productivity growth after the GFC.

The monetary and fiscal authorities responded to these processes in the following ways:

- After the dot-com crash the Federal Reserve temporarily lowered interest rates. These were increased during the peak of the housing boom and reduced to near-zero during the GFC. The Fed started lifting them again in 2015. The changes to monetary policy before 2001 have not been included in the simulations (Figure 8).
- The government provided a fiscal stimulus during the GFC by increasing government expenditures; this coincided with a temporary increase in the defence spending to GDP ratio. These processes temporarily reversed the long-term trend in the reduction of the government spending to GDP ratio (Figure 26).

The following scenarios have been simulated in order to demonstrate how the multiple processes overlapped and which elements in the government response were actually working:

- Historical changes in exogenous parameters without simulating the dot-com and real estate bubbles and the GFC ("HistoricalNoBubblesNoStimuli" scenario). This simulation also includes changes in high-income households' personal income tax rate.

- Historical changes in exogenous parameters with the dot-com and real estate bubbles and the GFC, without the monetary and fiscal stimuli (“HistoricalNoStimuli” scenario).
- Historical changes in exogenous parameters with the dot-com and real estate bubbles and the GFC, with the monetary stimulus but without the fiscal stimulus (“HistoricalNoFiscalStimulus” scenario). The introduction of the monetary stimulus was required to avoid a deep price deflation. Without reducing the interest rate in 2007-2008, the recession is so severe that the model shows a prolonged period of no investment, leading to lack of productivity growth. The results of this simulation may be inaccurate as there is no macroeconomic data to calibrate the dynamic behaviour of the model, when the system is no longer linear. The severity of the depression experienced by the simulated economy in this scenario gives support to the view that the monetary policy of the Federal Reserve saved the economy from an even worse fate than the GFC (Bernanke, 2010).
- Historical changes in exogenous parameters with the dot-com and real estate bubbles and the GFC, with the monetary and fiscal stimuli (“HistoricalGFC” scenario).

The graphs provided below illustrate how changes in individual parameters have contributed to the final trajectory of the GDP and unemployment rate. The dividends to firm profits ratio “ψ_D” affects the share of profits in the GDP (Figure 65). Another exogenous variable affecting the share of profits is the main wage expectations parameter “ω_0” (Figure 66). Together these parameters affect the rate of inflation for a given rate of unemployment (they shift the short-run Phillips curve). Changes in the fraction of the total demand for labour which is satisfied by high-income households “HILD” (affecting the distribution of disposable income between low-income and high-income households) are shown in Figure 67. The magnitudes of the changes in the share of profits in the GDP and in the fraction of the total demand for labour which is satisfied by high-income households have been calibrated so that the resulting high income households share in disposable income (Figure 88) has changed about 6% between 1984 and 2019. The changes in this parameter mimic in their magnitude the actual trajectory. The actual historical data depicted in Figure 12 has been also presented in Figure 88.

Changes in the target government expenditures to GDP ratio, determining fiscal policy in the long run are shown in Figure 68. Changes in average net import propensity are shown in Figure 69. Changes in the size of the labour force to total population ratio "LF_N" have been calibrated based on the actual labour force data (Figure 70). Labour supply in all the historical scenarios has been depicted in Figure 84; labour demand is determined endogenously by the economy. The trajectory of labour supply, based on the actual data, has been calibrated to produce a realistic unemployment rate trajectory. Participation rate is in fact also an endogenous variable but modelling this falls outside of the current study. The simulated labour supply trajectory diverts from the actual data after 2009 due to the crude calibration of productivity growth in the model (Figure 5 and Figure 87).

Changes in high-income households' personal income tax rate "Θ_{hh}" have been calibrated so that the model is stable, but the actual reduction of the average high-income tax rate is also reflected (Figure 71).

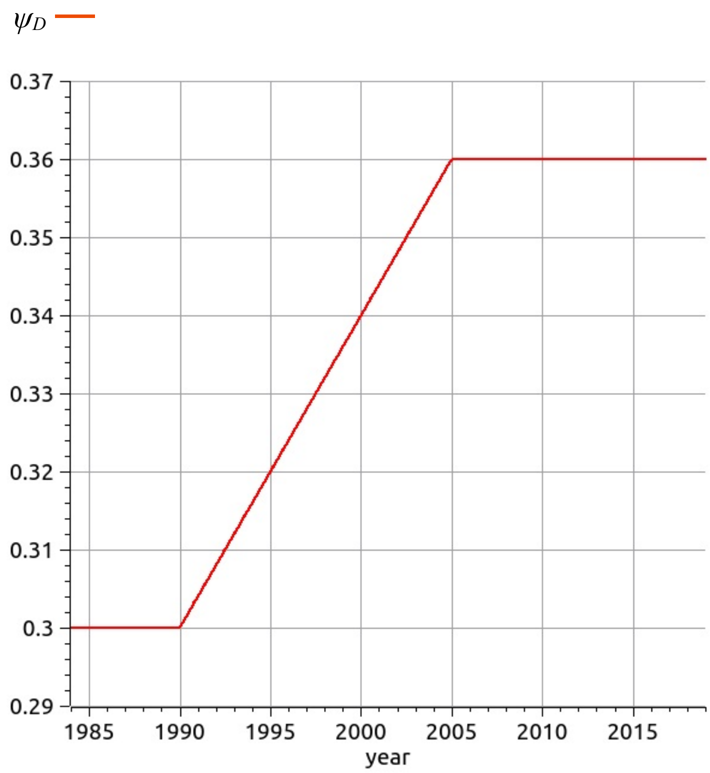

Figure 65: Historical scenarios, dividend to profit target ratio.

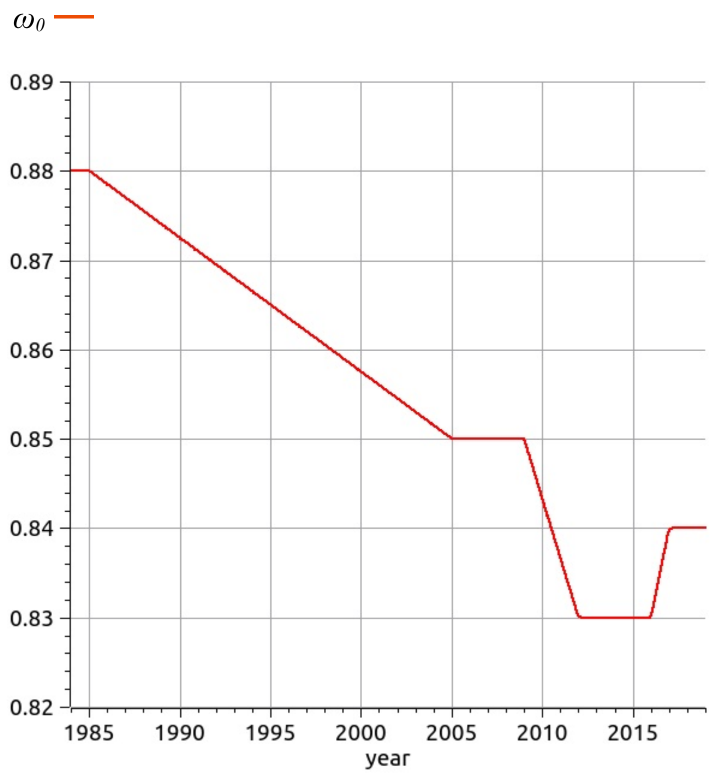

Figure 66: Historical scenarios, main wage expectations parameter.

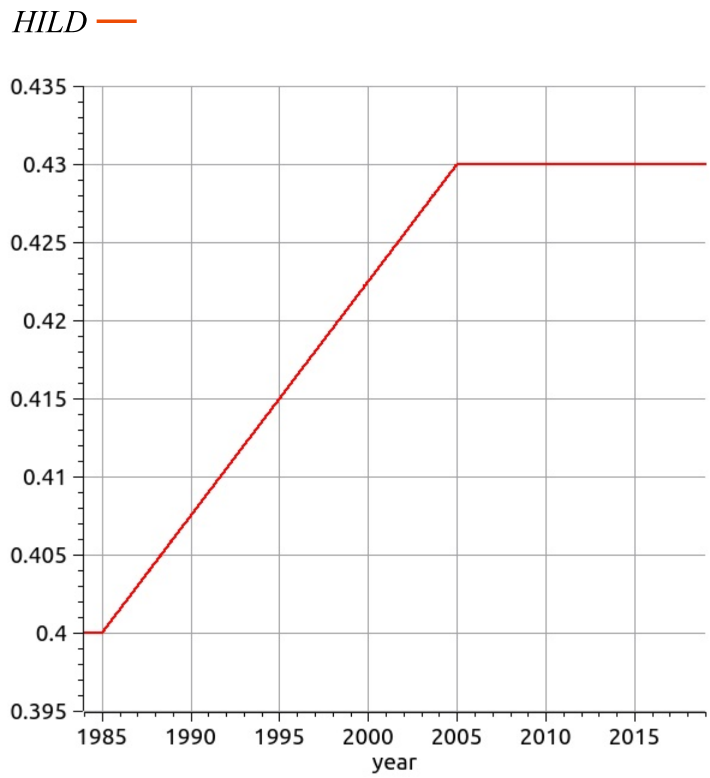

Figure 67: Historical scenarios, labour demand satisfied by high-income households.

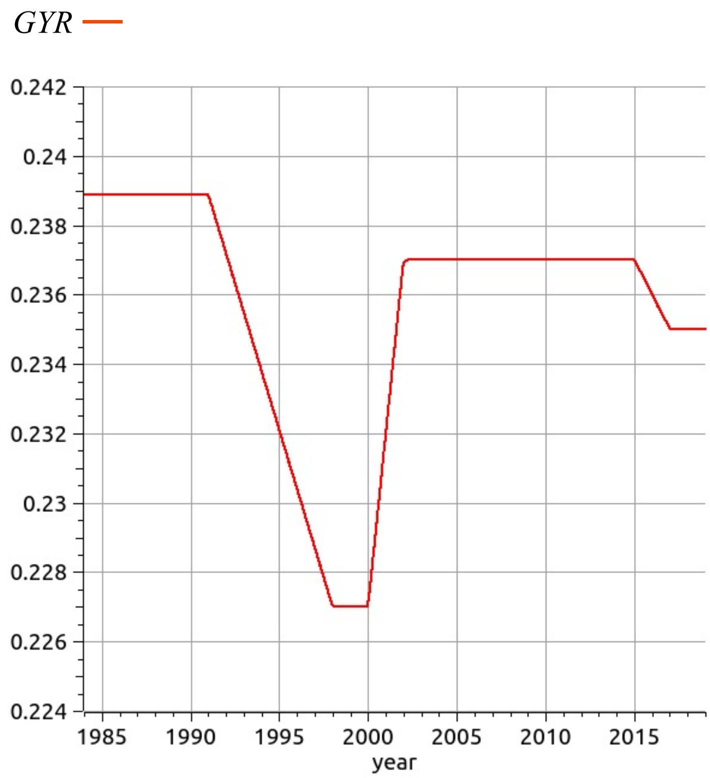

Figure 68: Historical scenarios, target government expenditures to GDP ratio.

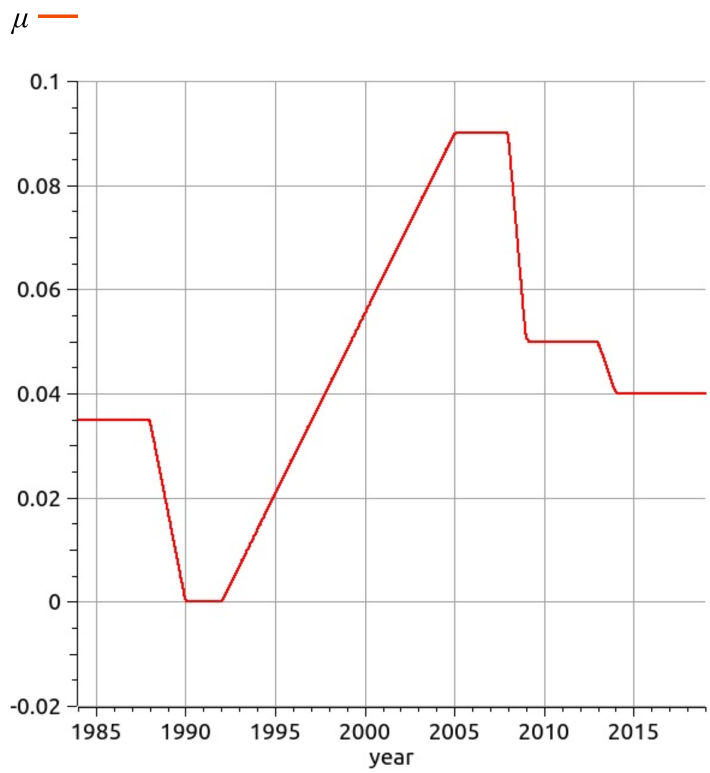

Figure 69: Historical scenarios, average net import propensity.

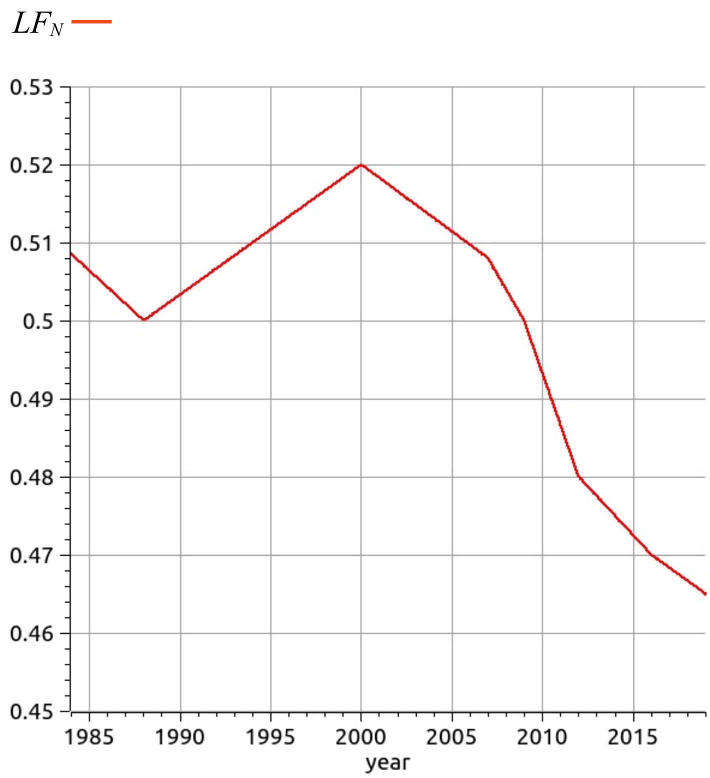

Figure 70: Historical scenarios, labour force to total population ratio.

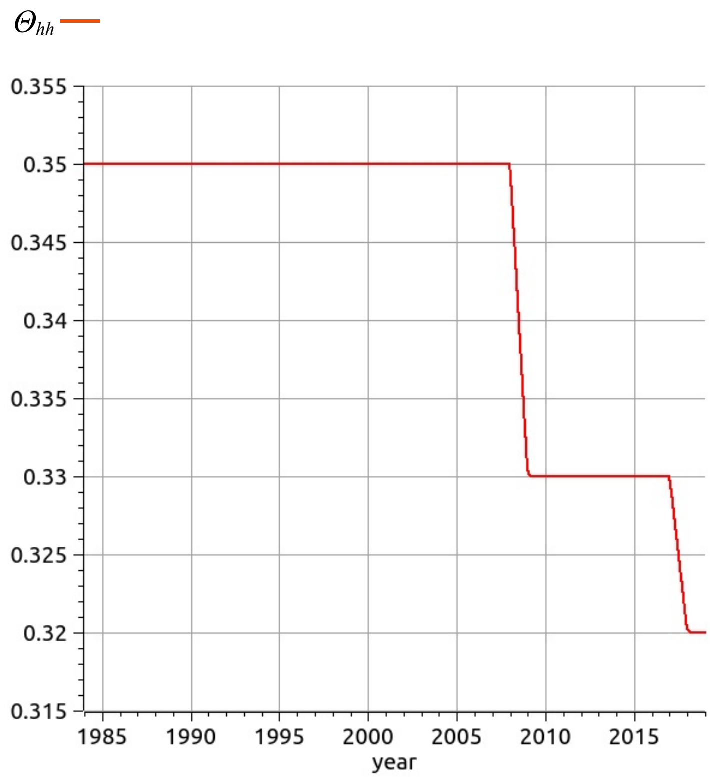

Figure 71: Historical scenarios, high-income household tax rate.

The following exogenous parameters have been changed in the “HistoricalNoStimuli”, “HistoricalNoFiscalStimulus” and “HistoricalGFC” scenarios:

- equities portfolio choice parameter “λ_{40}” (to simulate the dot-com boom) – shown in Figure 72
- target capital utilisation ratio “u_0” (to simulate the dot-com boom and the boom preceding the onset of the GFC) – shown in Figure 73
- new mortgage lending to total disposable income ratio “η” – shown in Figure 74, the impact of this parameter on net mortgage lending is depicted in Figure 76
- real estate value to mortgage debt stock ratio “REMOR” – shown in Figure 75, the impact of this parameter on the total value of real estate is depicted in Figure 77

Additionally the following exogenous parameter has been changed in “HistoricalNoFiscalStimulus” and “HistoricalGFC” scenarios:

- deposit interest rate “r_m” – shown in Figure 78

The changes in the equities portfolio choice parameter "λ_{40}" correspond to the changes in the relative value of equities to the total wealth of households which occurred during and after the dot-com bubble. Another parameter which was modified to simulate the bubble was the target capital utilisation rate "u_0" as companies issuing new equities were willing to invest more than usual on productive (fixed) capital. There was also a small temporary increase in corporate investment just before the GFC, this increase in corporate investment filled up the demand gap after the end of the actual housing bubble in 2006 as depicted in Figure 3. The corporate investment to GDP ratio trajectory assumed in the "HistoricalGFC" scenario is shown in Figure 83.

Changes in the monetary policy (the monetary stimuli introduced in 2001 and 2008) are included in the "HistoricalNoFiscalStimulus" and "HistoricalGFC" scenarios, the changes in the deposit rate of interest are shown in Figure 78.

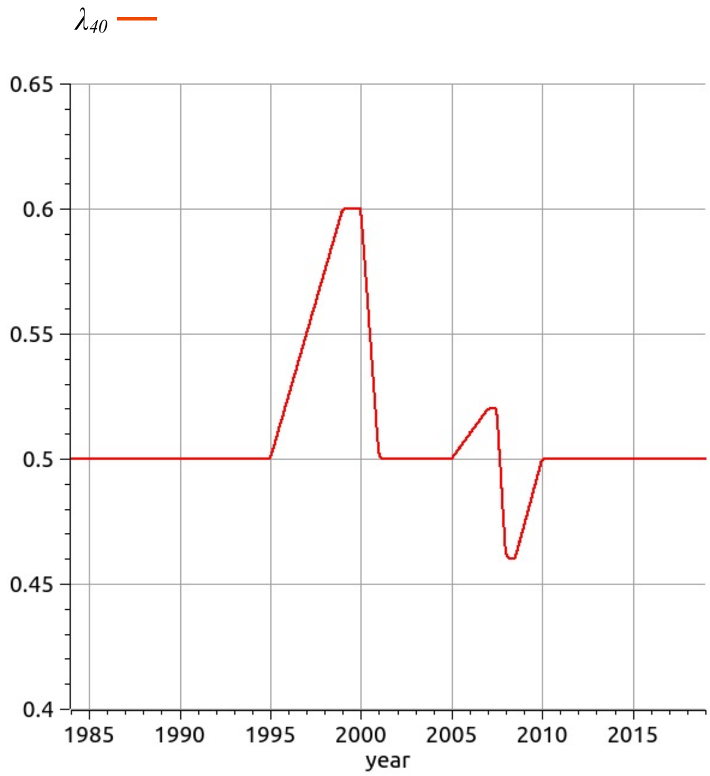

Figure 72: Historical scenarios, equities portfolio choice parameter.

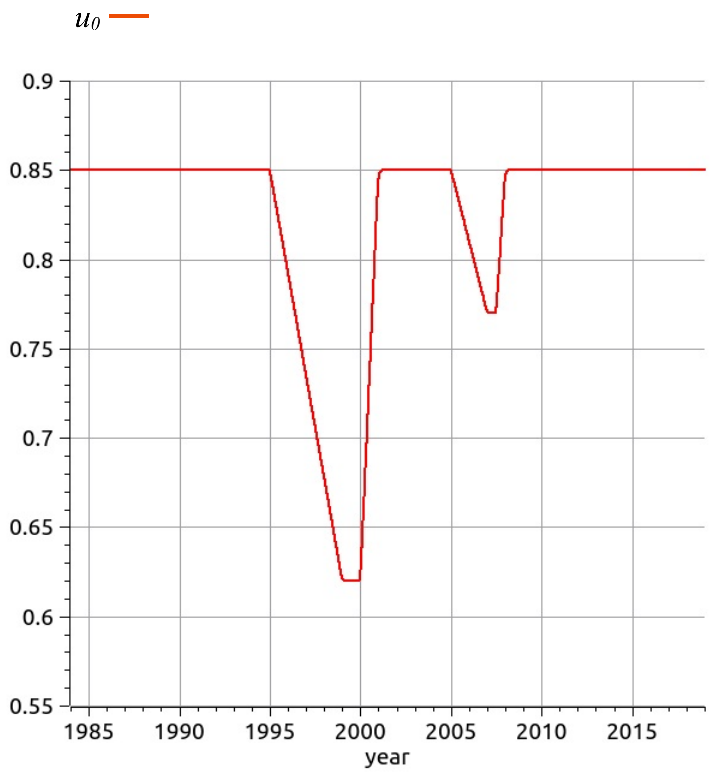

Figure 73: Historical scenarios, target capital utilisation rate.

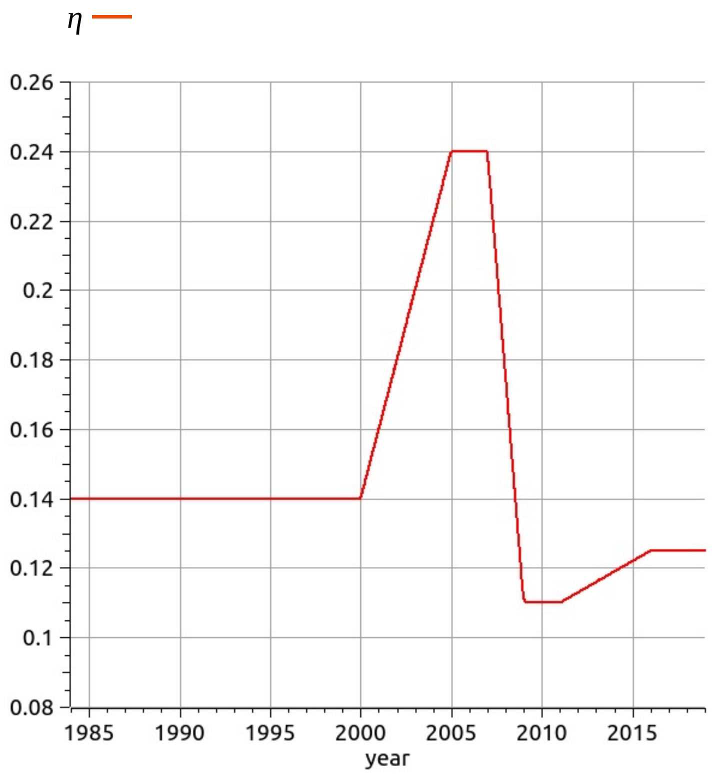

Figure 74: Historical scenarios, new lending parameter.

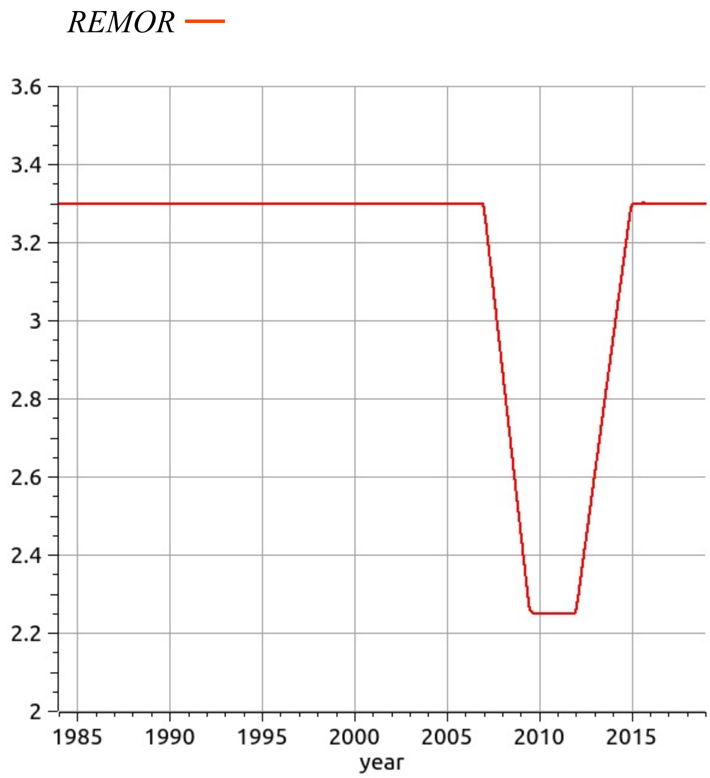

Figure 75: Historical scenarios, relative value of real estate.

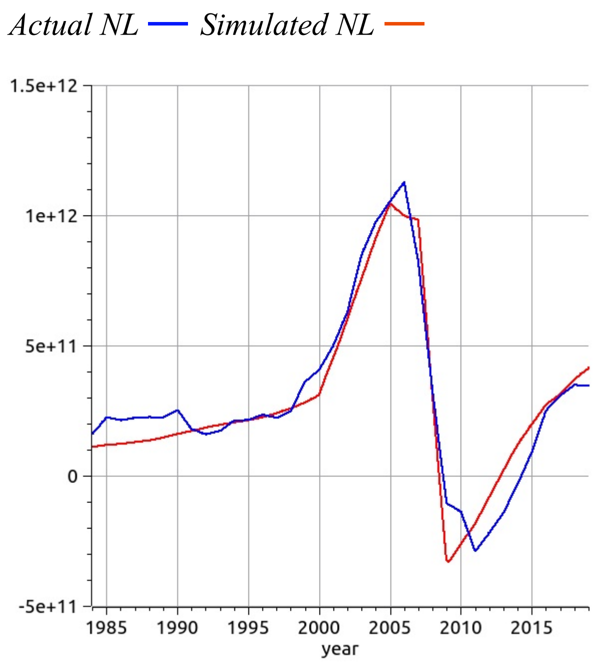

Figure 76: Historical scenarios, net mortgage lending.

Based on the data from Federal Reserve Economic Data (CONSUMER, HMSDODNS).

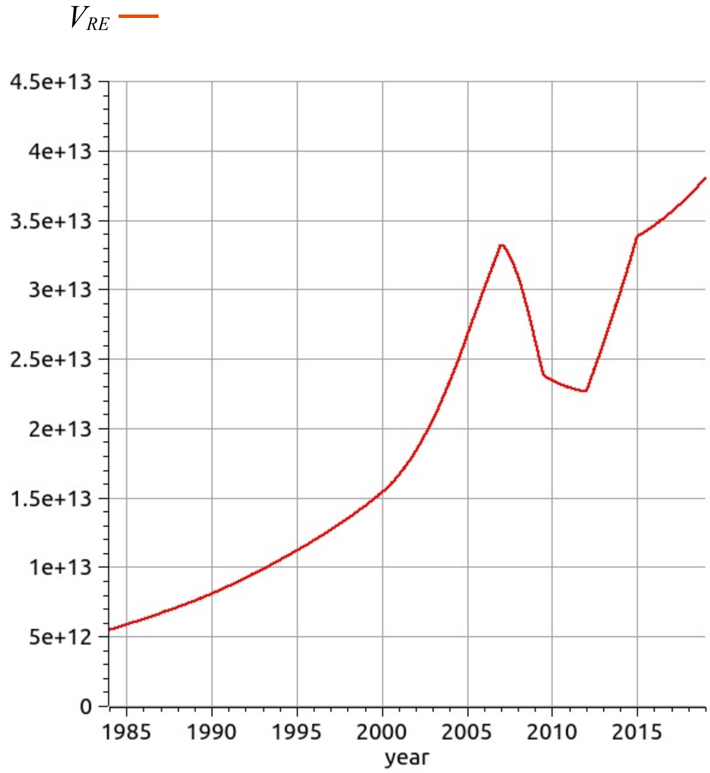

Figure 77: Historical scenarios, impact on total real estate value.

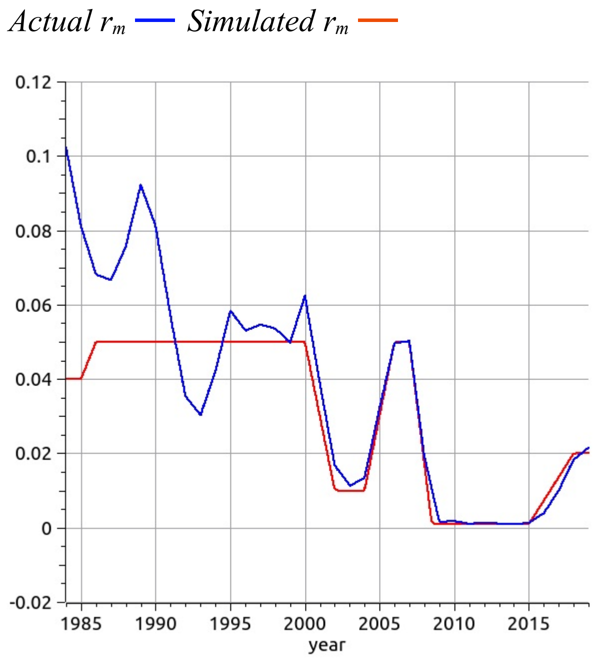

Figure 78: Historical scenarios, rate of interest.

Based on the data from Federal Reserve Economic Data, Board of Governors of the Federal Reserve System (US), (FEDFUNDS), A fiscal stimulus has been added in the “HistoricalGFC” scenario which has been designed to simulate the actual historical trajectories of GDP and unemployment. The stimulus has been implemented by changing the rate of growth of government expenditures, “gr_g” (Figure 79). This generates a short term spike in government expenditures (shown in Figure 80) however the long run trajectory is still mostly determined by changes in the long-run target ratio “GYR” (Figure 68). The control parameters (“gr_g” and “GYR”) have been calibrated so that the resulting simulated trajectory of the government expenditures to GDP ratio resembles the actual trajectory with an offset, required to compensate for the simplicity of the model.

The budget deficit to GDP ratio is partially endogenous as government revenue depends on the taxes which indirectly depend on the GDP (as shown in Figure 81). The actual trajectory (depicted in Figure 9) has not been simulated correctly, especially in the initial period 1984-2000, due to highly simplified taxation schedule and crude model of the distribution of gross income among social groups. Short-term changes in the budget deficit to GDP ratio after 2000 have been reproduced more correctly, the stimulus largely “pays for itself” due to increased tax revenue and lower social transfer payments.

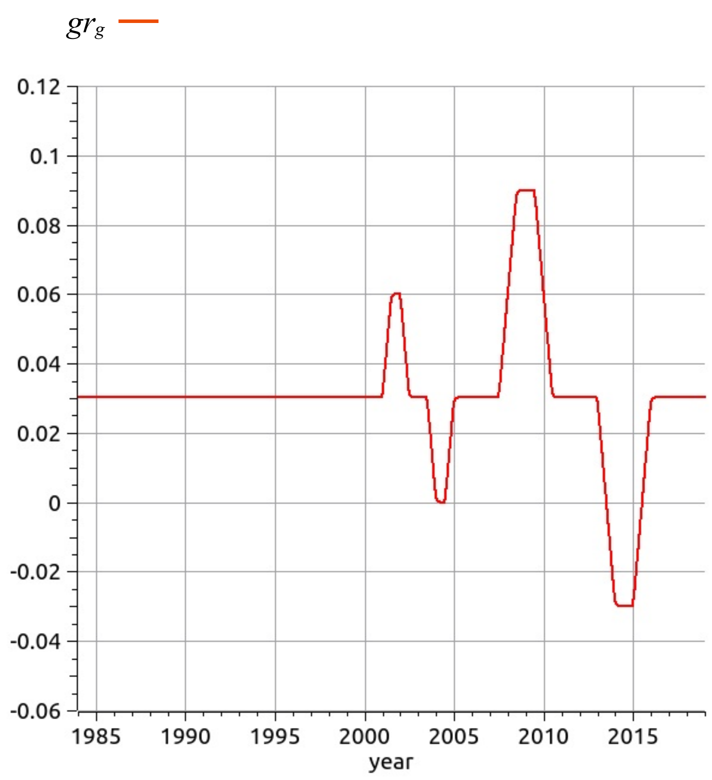

Figure 79: "HistoricalGFC" scenario, rate of growth of government expenditures.

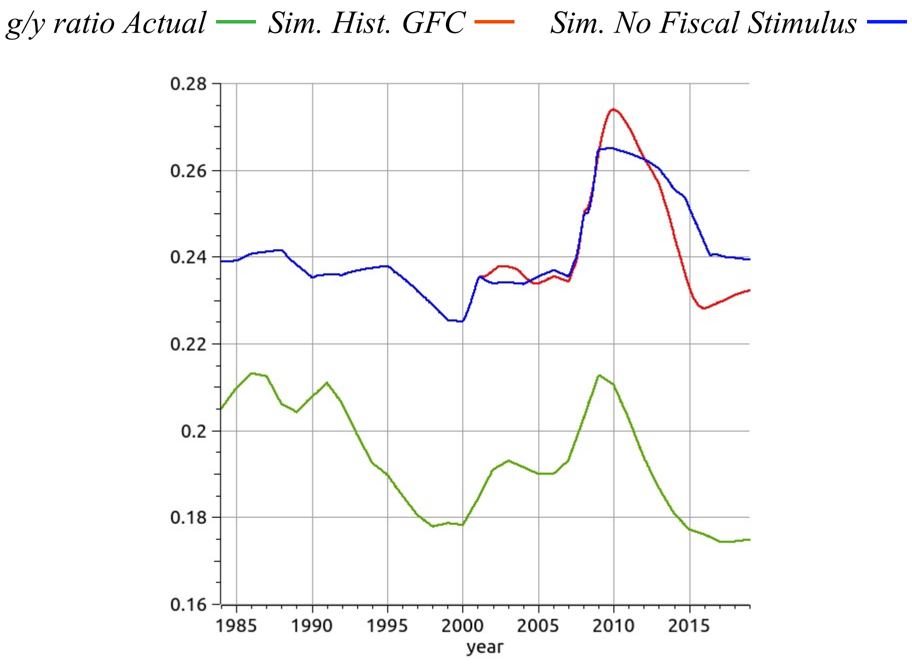

Figure 80: Historical scenarios, government expenditure to GDP ratios.

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (GCEA, GDPA)

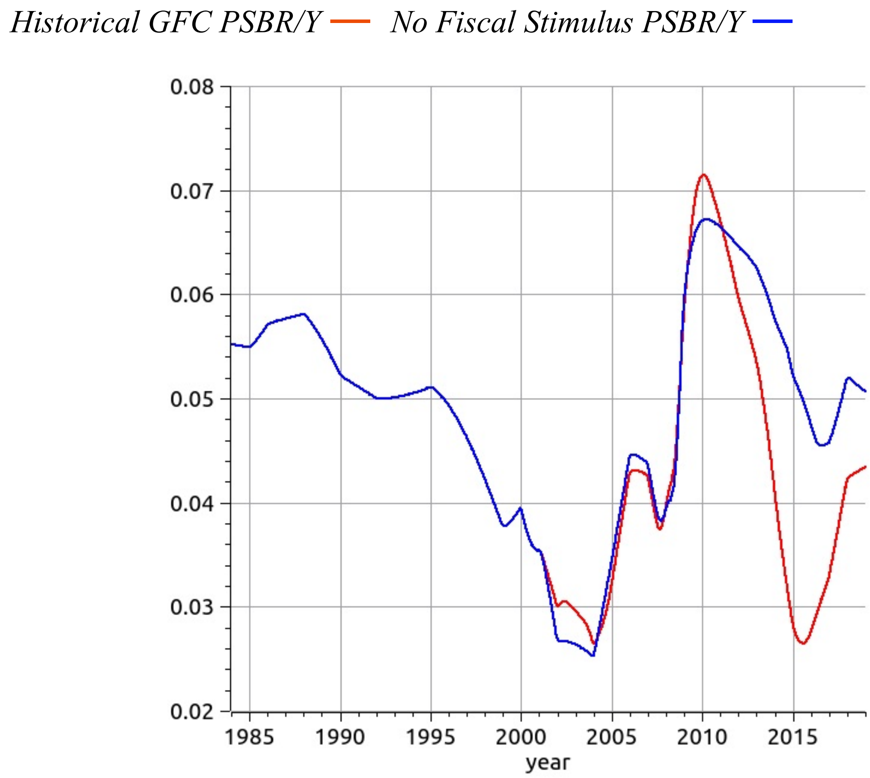

Figure 81: Historical scenarios, budget deficit to GDP ratios.

The changes in the trade balance, which is mainly driven by a control variable "μ" have been shown in Figure 82. Trade balance significantly affects the domestic aggregate demand.

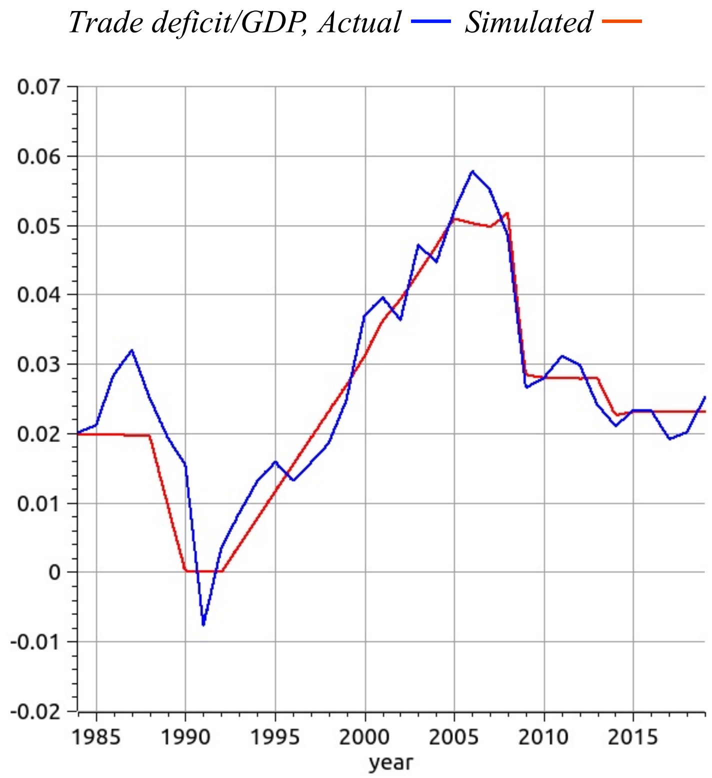

Figure 82: "HistoricalGFC" scenario, trade deficit as a fraction of GDP. Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (NETFI, GDP)

The ratios of corporate, residential and total private investment total to GDP in the model have been shown in Figure 83. These (mostly exogenous) trajectories can be compared with the actual data presented in Figure 3. The simulated data does not reflect first the dip in the total investment in the early 1990s related to the mild recession of 1990-91 and the second dip which happened after the bursting of the dot-com bubble and before the housing boom. The ratio of corporate investment to GDP in the model is offset by a constant value, due to imperfect model calibration. However the overall timing and magnitude of the changes (about 5% of the GDP during the GFC) has been preserved in the model.

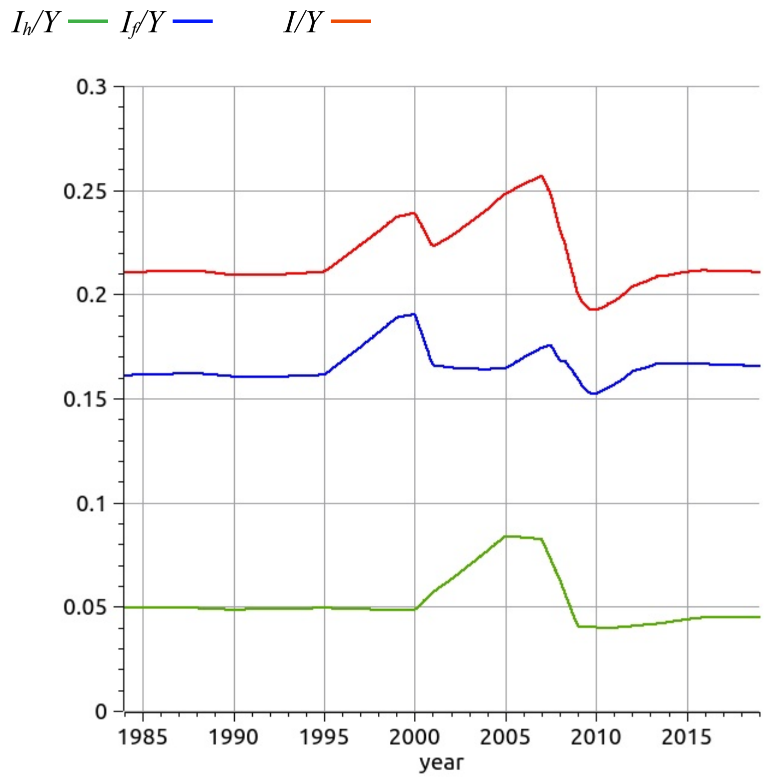

Figure 83: "HistoricalGFC" scenario, investment to GDP ratios.

The following graphs illustrate the responses of the simulated system to the changes in exogenous parameters. The drop in employment (Figure 85) slows down productivity growth. This impacts the GDP trajectory (Figure 91). Another parameter affecting the GDP trajectory in the log run is the share of high income households in the total disposable income of the households (Figure 88).

Productivity growth (Figure 86 and Figure 87) has been partially endogenised in the model and it is progressively reduced when unemployment exceeds a threshold value. It is the loss of actual productivity compared with the potential (trend) productivity growth trajectory which eventually reduces the unemployment rate when aggregate demand is not rising at the pace required to consume all that can be produced. The model of the labour market is however too simplified to correctly account for the changes in participation rate and productivity which would generate a stable relationship between aggregate demand and unemployment, known as Okun's law (as already mentioned, the participation rate is considered in the model to be an exogenous parameter). The adverse drop in productivity can be seen in the “HistoricalNoBubbles NoStimulus” scenario, due to the reduction in government expenditures as a fraction of GDP and the distributional changes. The bubbles have postponed the emergence of the “Secular Stagnation”.

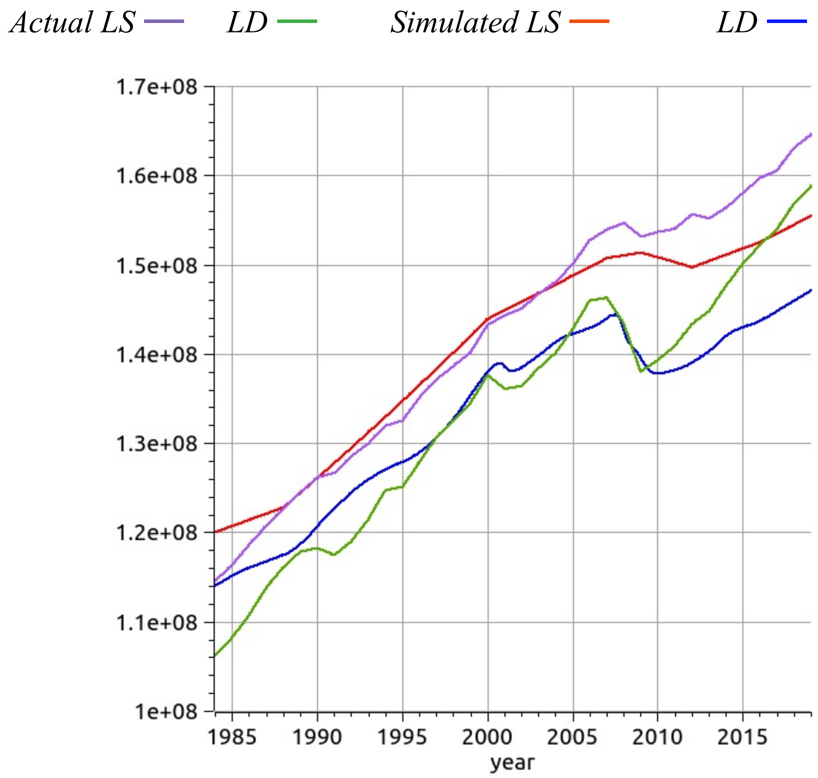

Figure 84: “HistoricalGFC” scenario, labour supply and demand.
Based on the data from Federal Reserve Economic Data, U.S. Bureau of Labor Statistics (CLF16OV, CE16OV),

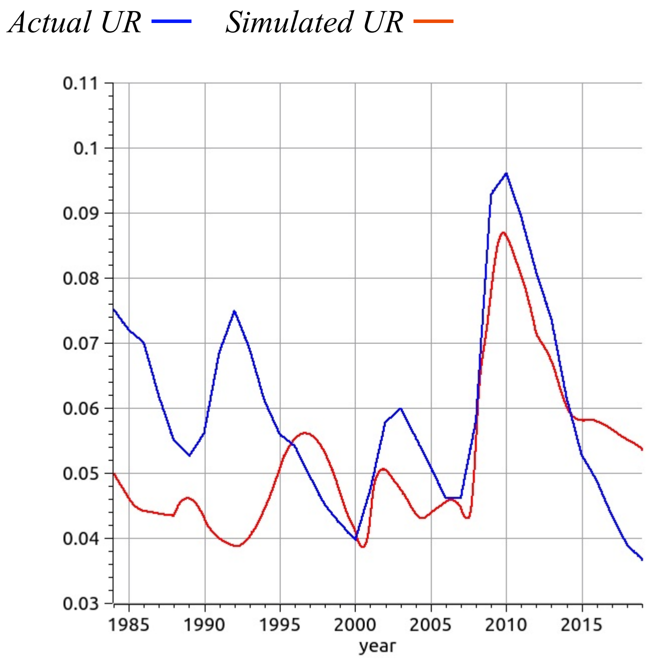

Figure 85: "HistoricalGFC" scenario, rate of unemployment.
Based on the data from Federal Reserve Economic Data, U.S. Bureau of Labor Statistics (UNRATE)

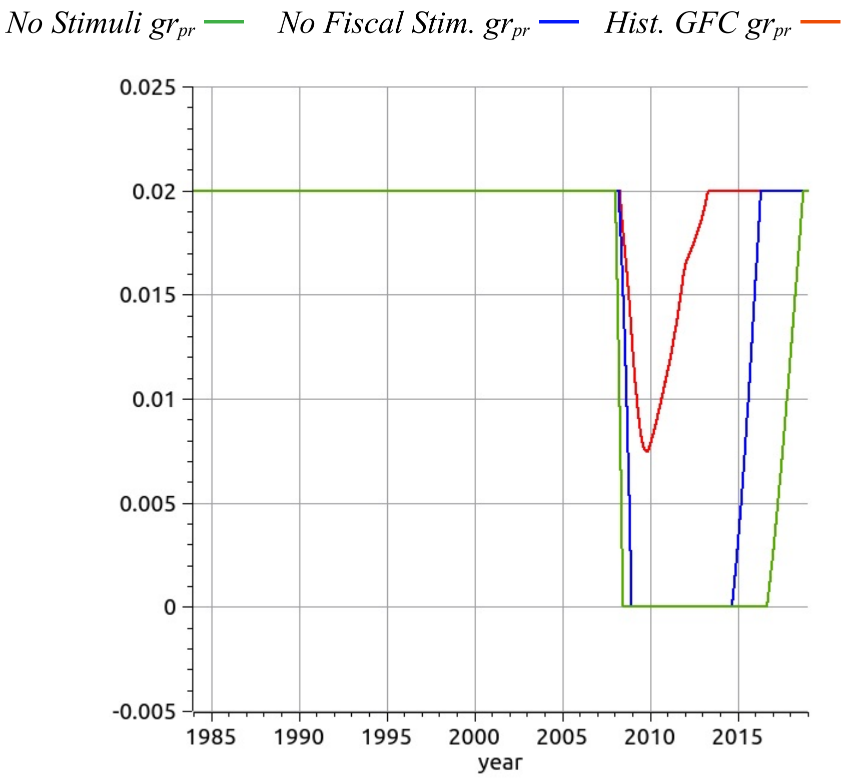

Figure 86: Historical scenarios, rate of growth of labour productivity.

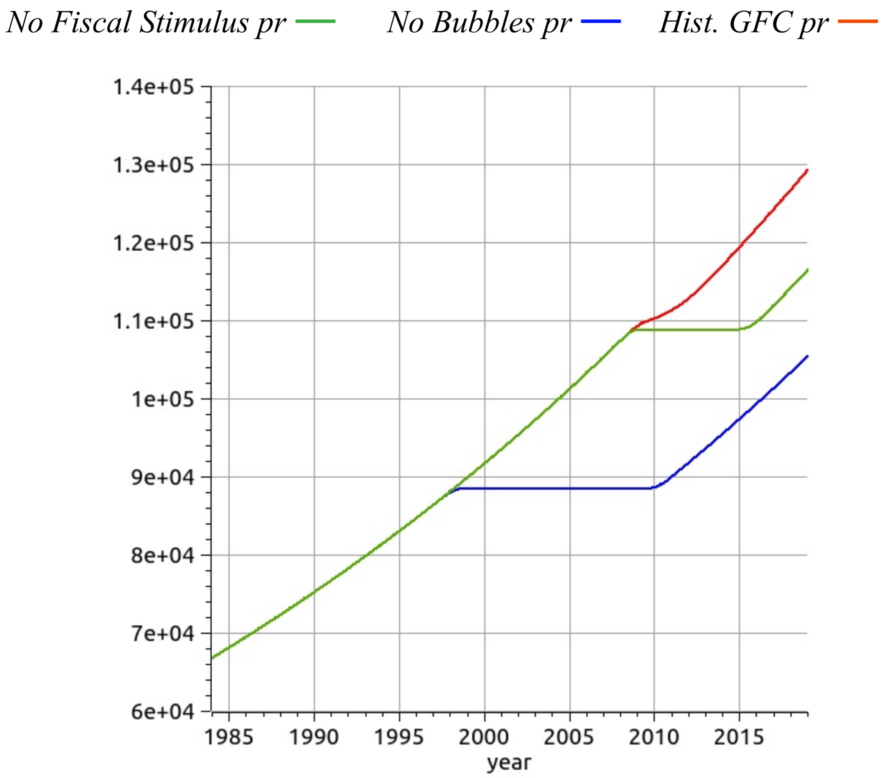

Figure 87: Historical scenarios, labour productivity.

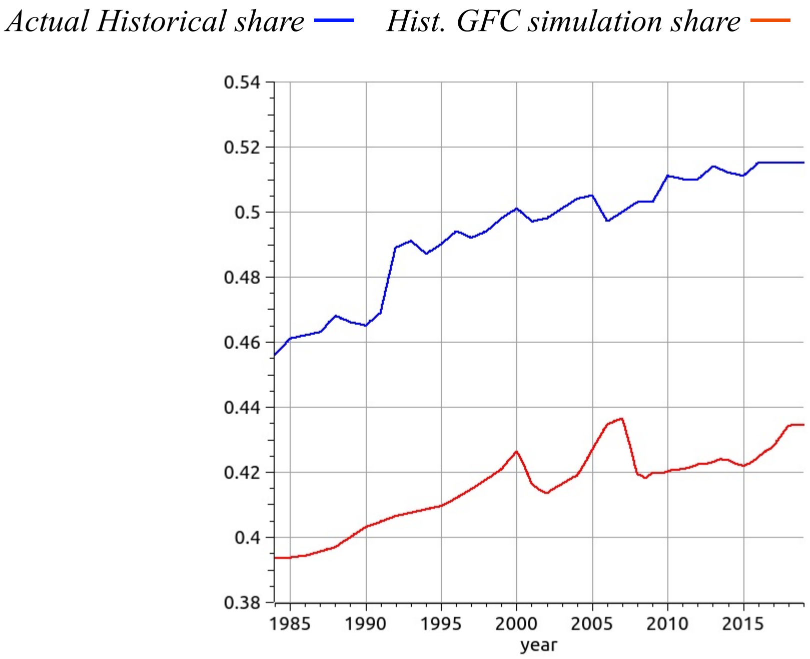

Figure 88: Actual and simulated "HistoricalGFC" scenario, share of high income households in total disposable income.

Based on the data from World Inequality Database (USA, sdiinc992j, p80p100)

The recession which followed the Great Financial Crisis would be much deeper without the monetary and fiscal stimuli. According to the model, the economy would have experienced price deflation, as shown in Figure 89.

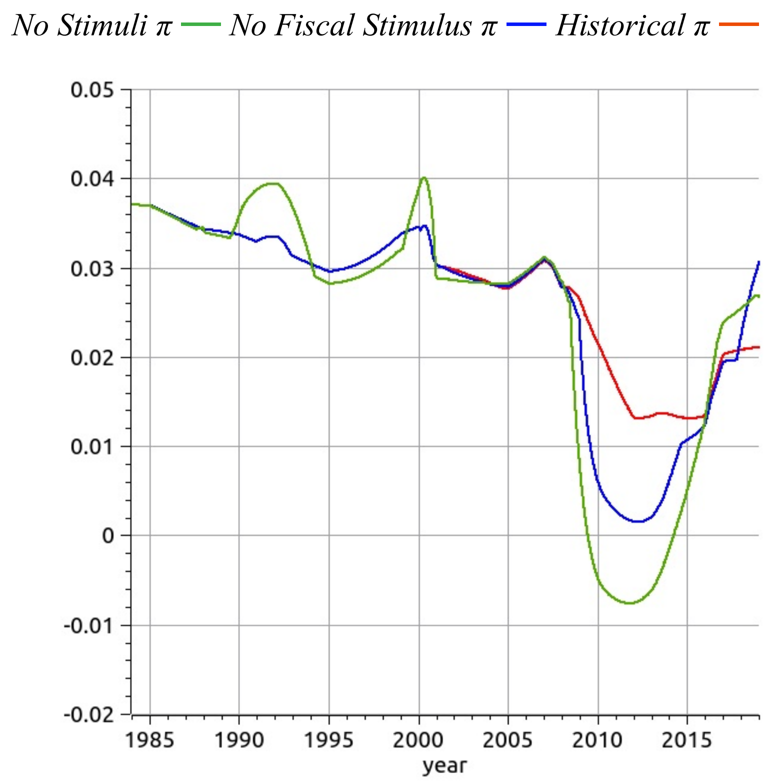

Figure 89: Historical scenarios, price inflation.

The final graphs are depicting the GDP trajectory (Figure 90 and Figure 91). The main goal of the simulation was to reproduce the actual historic data. It can be seen how the dot-com and housing bubbles stimulated the economy in the period 1996-2006 (Figure 90). The positive impact of the monetary and fiscal stimuli is also clearly visible in Figure 91 however it can be claimed that the size of the fiscal stimulus was too small to prevent productivity losses caused by an increase in the unemployment rate. The actual historical data depicted in Figure 1 has also been presented in Figure 91. Further discussion of the simulation results is provided in Section 4.16.

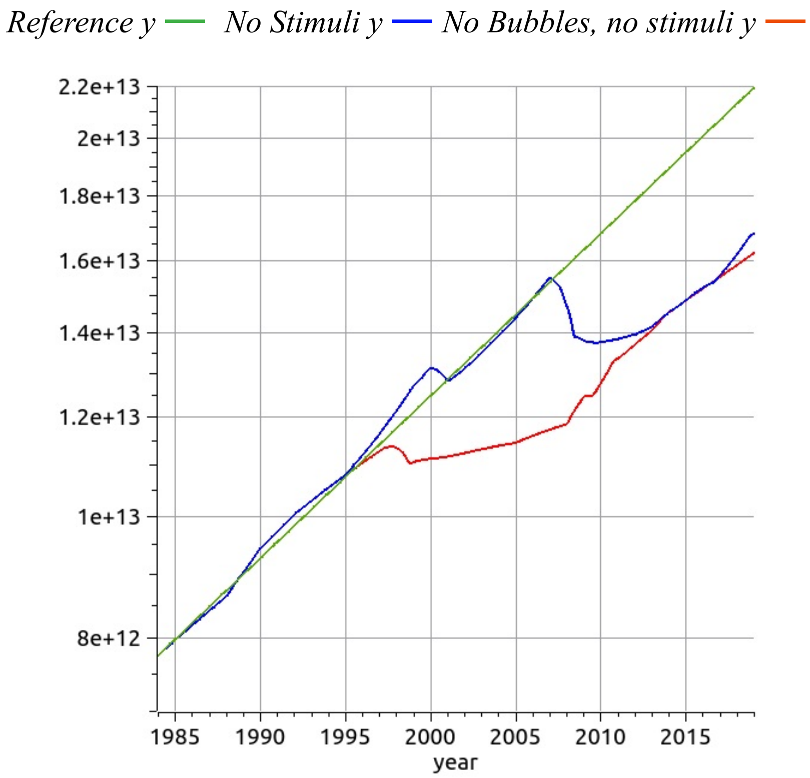

Figure 90: Historical scenarios (no stimuli), real GDP trajectories.

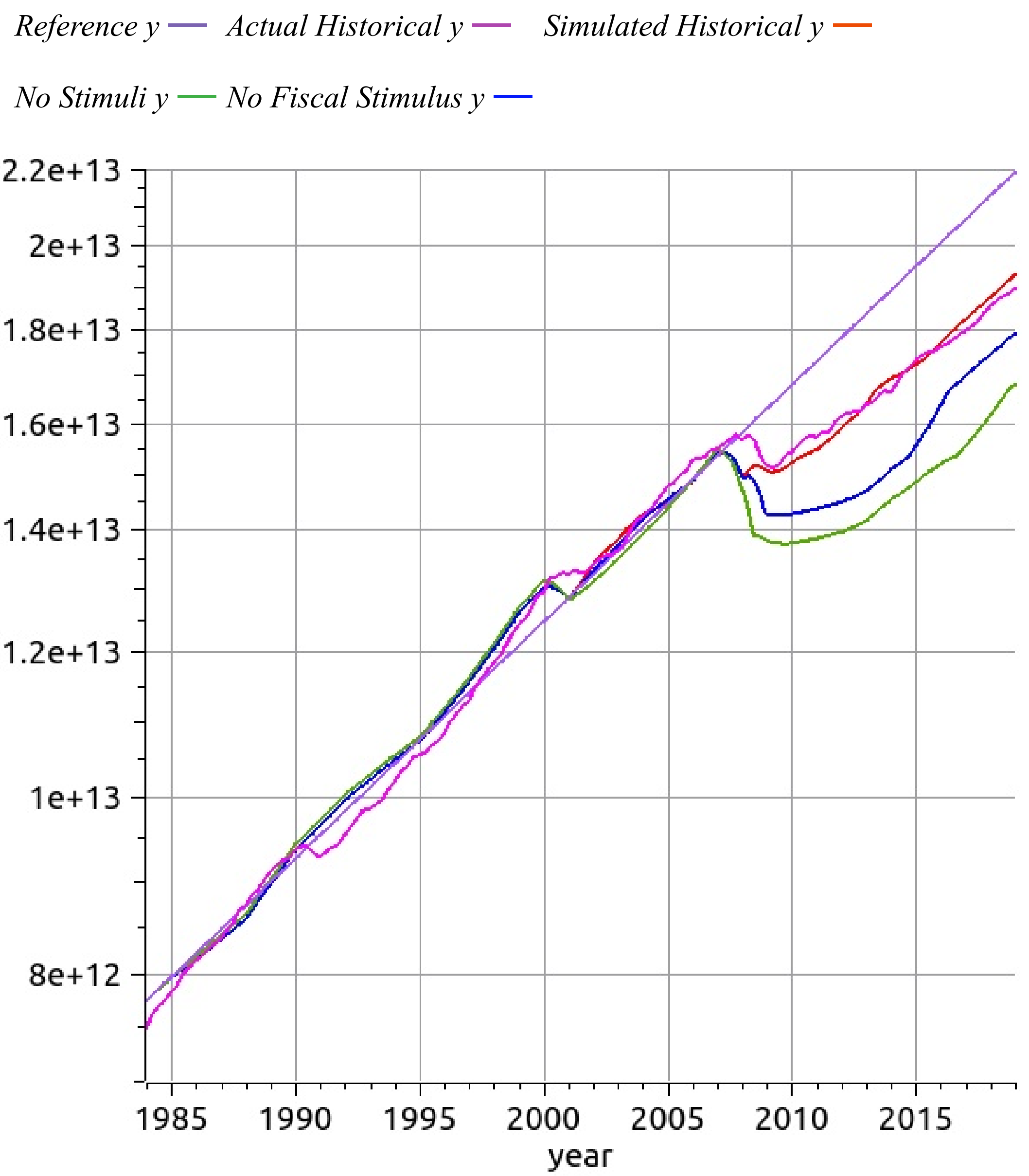

Figure 91: Actual and simulated historical scenarios, real GDP trajectory

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (GDPC1).

4.14 Simulation of long-run recovery scenarios

The goal of the simulations is to examine the policies which are available to overcome "Secular Stagnation" in the long run. The baseline for these simulations is the "HistoricalGFC" scenario, simulating the processes developing in the American economy before 2019. Some adjustment of the fiscal policy and wage expectations was required in 2019 in order to avoid oscillations in the trajectories. After 2020, all the parameters used to simulate the "Historical" scenario remain unchanged. The following policies have been simulated:

1. A persistent fiscal stimulus implemented by increasing the long-run target government expenditures to GDP ratio "GYR" (Figure 92). The scenario is called "Fiscal Expansion".
2. An income redistribution policy implemented by decreasing the personal income tax rate for low-income households "Θ_{hl}" (Figure 93) and increasing personal income tax rate for high-income households "Θ_{hh}" (Figure 94), without changing the government deficit to GDP ratio. The scenario is called "Income Redistribution".

Both scenarios have been calibrated to deliver similar results, an increase in real GDP by about 3% compared with the continuation of the "HistoricalGFC" scenario (Figure 95) and a drop in the unemployment rate (Figure 96). A significantly larger increase in GDP and catching up with the "Reference" growth trajectory is not possible without assuming a higher rate of productivity growth (see Section 4.10).

The "fiscal expansion" and "income redistribution" scenarios differ in the fiscal outcome (budget deficit, Figure 97 and government debt to GDP ratio, Figure 98). Obviously, the measure of inequality in the distribution of disposable income (the Gini index) is different in these scenarios (Figure 99).

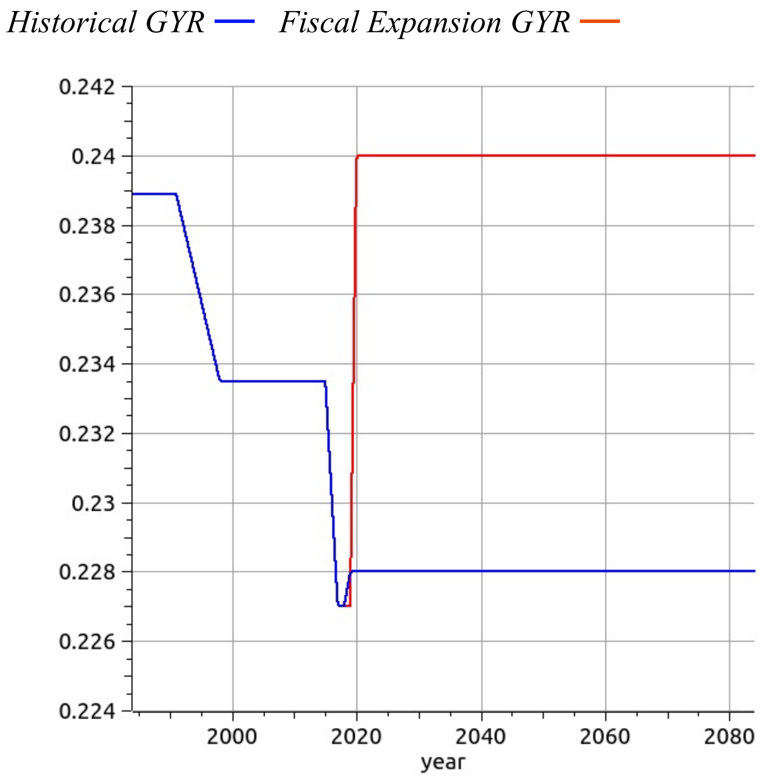

Figure 92: Long run recovery, target government expenditures to GDP ratios.

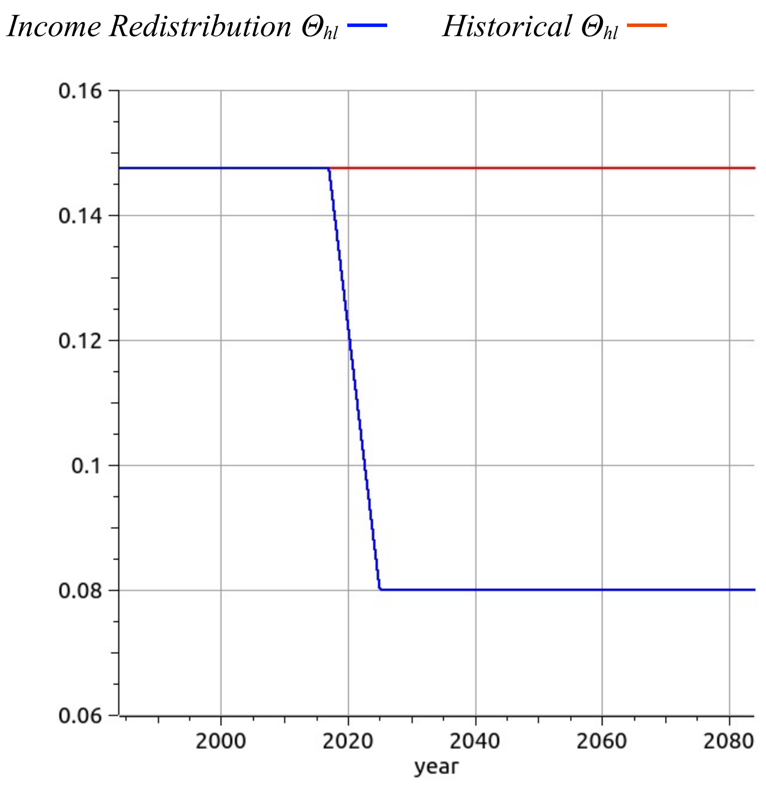

Figure 93: Long run recovery, low-income household tax rate .

Figure 94: Long run recovery, high-income household tax rate .

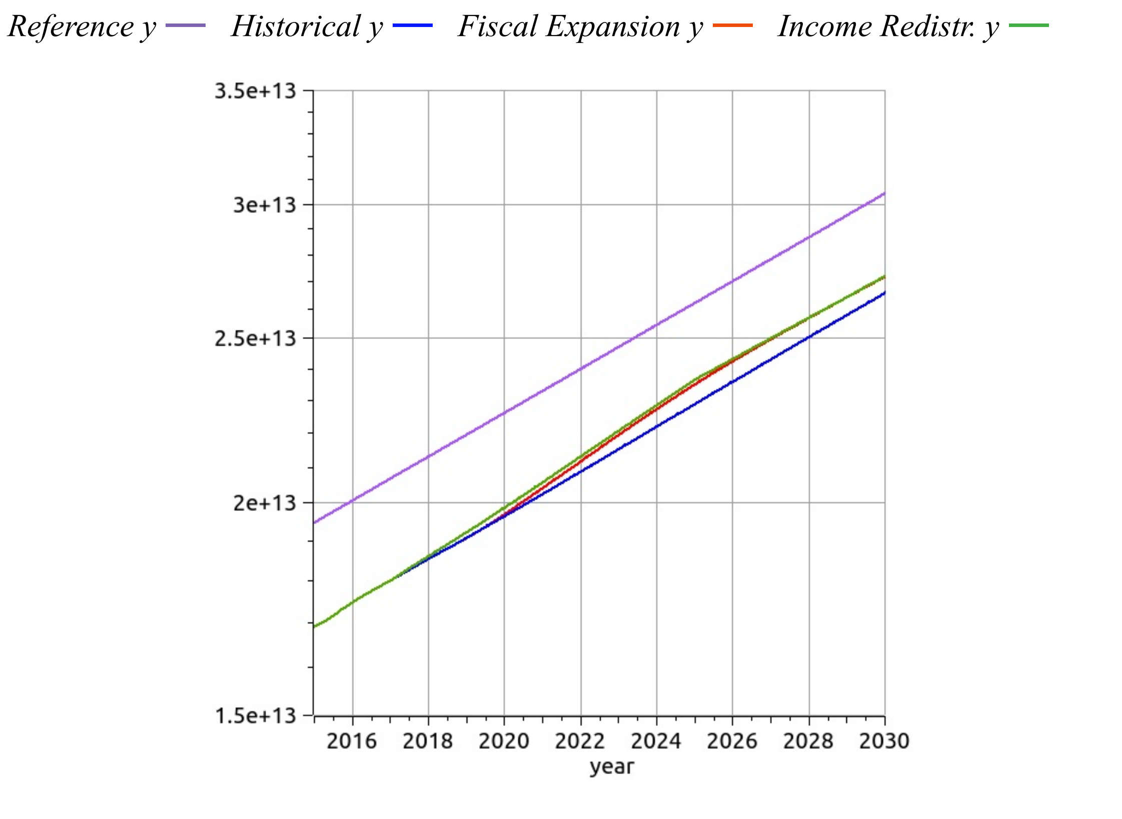

Figure 95: Long run recovery, impact on GDP

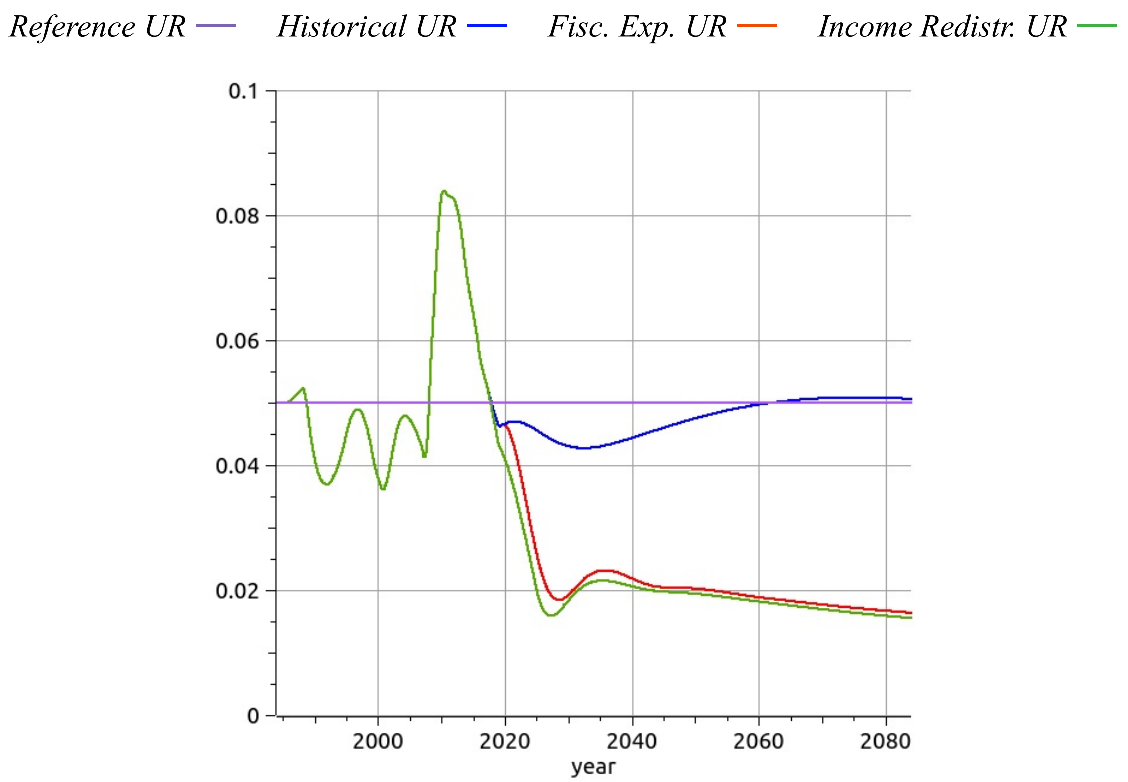

Figure 96: Long run recovery, impact on unemployment rate.

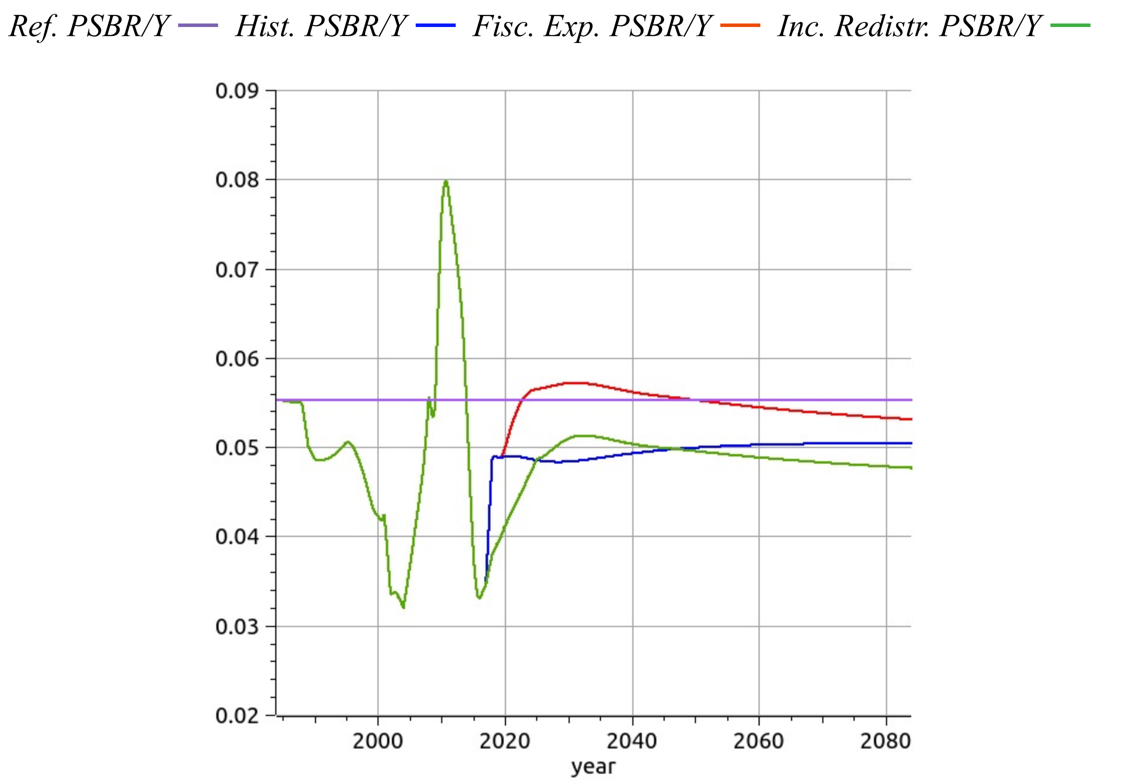

Figure 97: Long run recovery, impact on budget deficit.

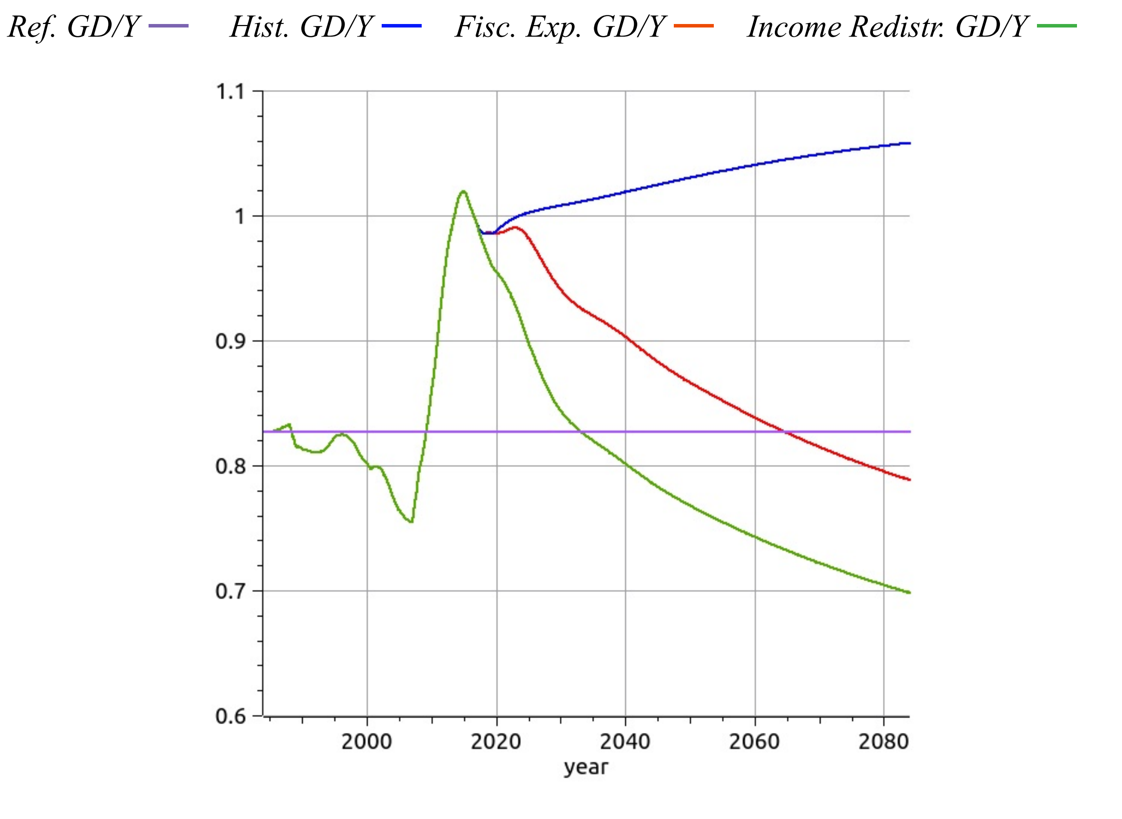

Figure 98: Long run recovery, impact on government debt to GDP ratio.

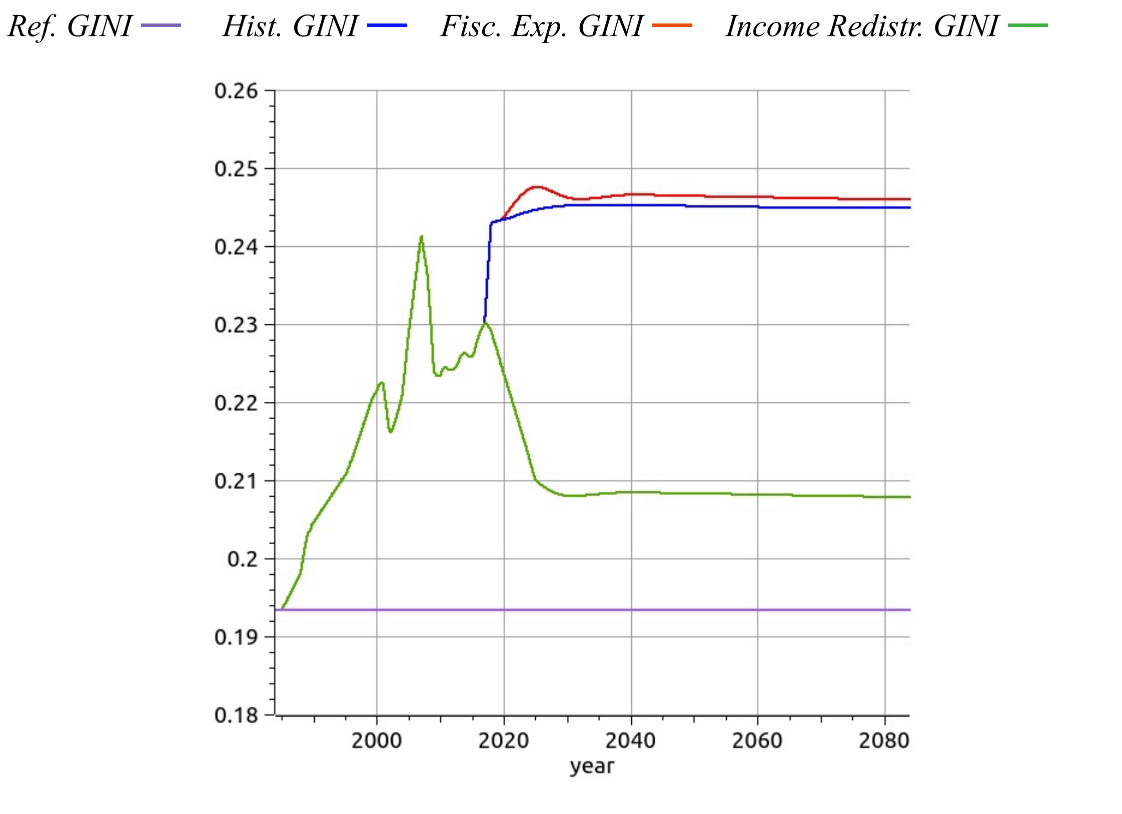

Figure 99: Long run recovery, impact on Gini coefficient.

4.15 The emergence of dynamic phenomena in the model

The following dynamic phenomena have been reproduced within the model:

- changes in the personal saving rate
- the credit impulse driving the growth of private demand
- a pseudo-Goodwin cycle
- the Gibson paradox

4.15.1 Changes in the personal saving rate

The personal saving rate is defined as:

$$PSAVERT(t)=\frac{YD_r(t)-C(t)}{YD_r(t)} \tag{166}$$

Let us assume in the short run the following simplified form of the consumption function (depicted in Figure 20):

$$C(t)=C_0+\alpha\cdot YD_r(t) \tag{167}$$

From (166) and (167) we get:

$$PSAVERT(t)=1-\alpha-\frac{C_0}{YD_r(t)} \tag{168}$$

In a model with a consumption function with constant coefficients, the personal saving rate is not constant. Changes are an artefact of the changes in the disposable income caused by exogenous shocks, such as the change in government spending, as shown in Figure 100 (generated using "FiscalStimulus" simulation scenario).

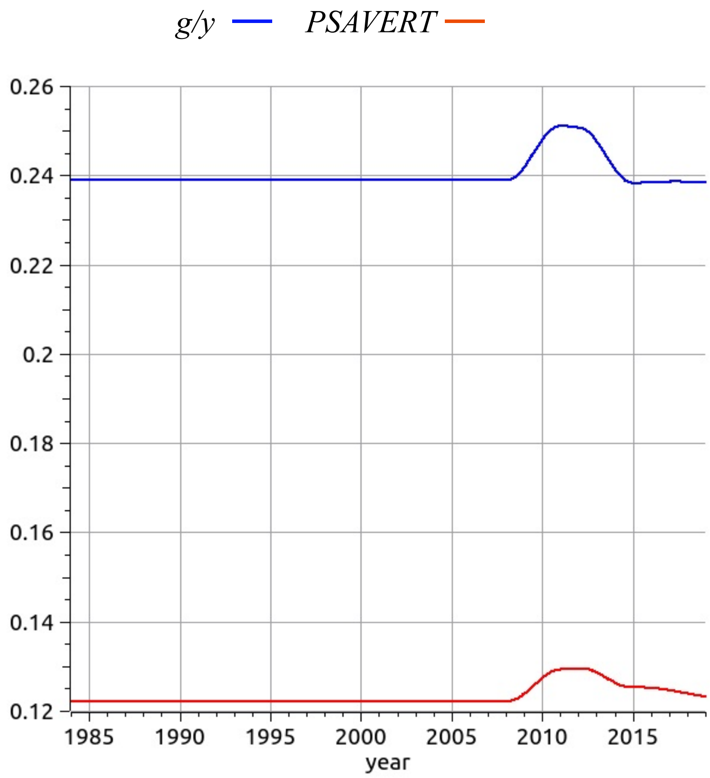

Figure 100: Impact on increased government spending on the personal saving rate.

The impact of changes in the rate of gross mortgage lending in the model is more complex as some of the borrowed money is spent on buying land from previous owners, who then increase their consumption. This financial flow is not included in the measures of disposable income. A reduction in the rate of gross mortgage lending usually does not affect the pace of loan repayment, which directly reduces consumption. This explains an increase in the rate of personal saving date during the debt deleveraging phase of the property cycle. These processes are illustrated in Figure 101 (generated using "HousingBubble" simulation scenario).

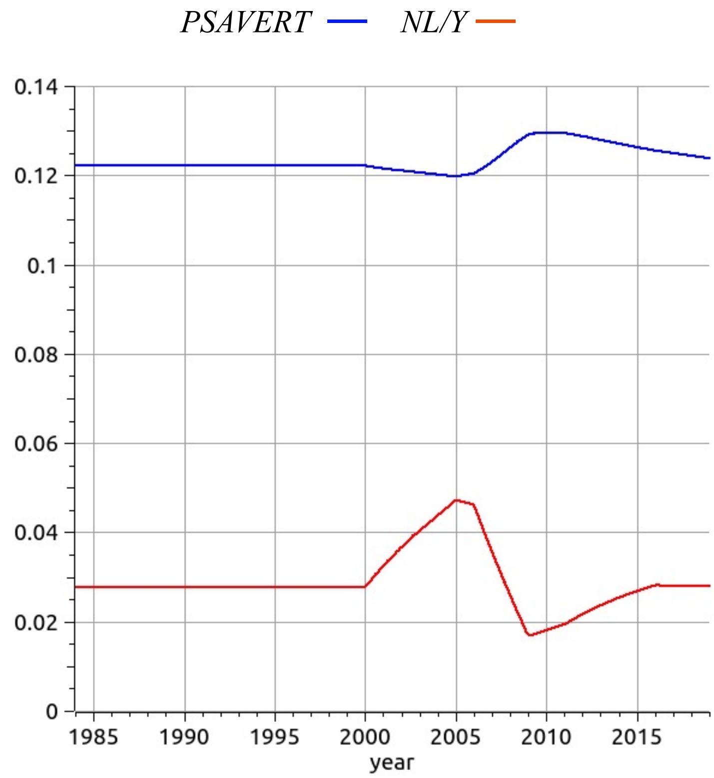

Figure 101: Impact on changes in mortgage lending on the personal saving rate.

It is obvious that in the real economy the coefficients used to define the consumption function used in the model are not constant and the function itself is only an imperfect approximation of the aggregate behaviour of the agents. Human consumers are impacted by the changes in “customer confidence” and by changing cultural trends. Nevertheless, the actual changes in the personal saving rate have been partially reproduced in the “HistoricalGFC” scenario. The personal saving rate was low during the housing boom but increased when households limited their net borrowing and when negative wealth effects lowered the consumption of high-income households. These effects can be seen in Figure 102. At least a fraction of the changes in the actual personal saving rate, which are usually attributed to the changes in the behaviour of the consumers (savers) can in fact be attributed to the changes in the behaviour of the borrowers and to other exogenous processes.

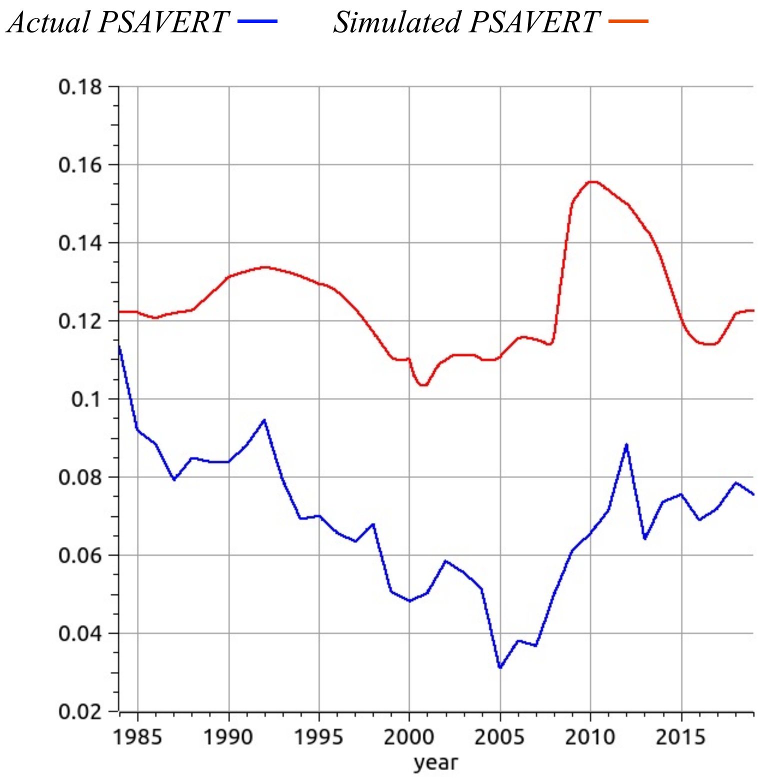

Figure 102: "HistoricalGFC" scenario, personal saving rate.

Based on the data from Federal Reserve Economic Data, U.S. Bureau of Economic Analysis (PSAVERT).

4.15.2 Credit Impulse and the growth of private spending

The idea that the credit impulse affects the growth of private spending has been presented by Biggs and Mayer (2013). The concept can be intuitively explained in the context of a simple Keynesian spending multiplier. A change in the volume of private sector debt-financed investment and consumption (a second derivative of the stock of loans divided by the nominal GDP), leads to (an amplified by the multiplier effect) change in the private sector demand (a derivative of the sum of consumption and investment, divided by the sum of consumption and investment).

Credit impulse is defined (in the model) as:

$$CREDIMP(t)=\frac{d\left\{\frac{d[L_f(t)+L_h(t)]}{dt}\cdot\frac{1}{Y(t)}\right\}}{dt} \tag{169}$$

Relative (detrended) change in the private sector demand is defined (in the model) as:

$$RPSG(t)=\frac{d[c(t)+i(t)]}{dt}\cdot\frac{1}{(c(t)+i(t))}-\{(1+gr_N)\cdot[1+gr_{pr}(t)]-1\} \tag{170}$$

The effect can be seen in its purest form in the “HousingBubble” scenario, where direct wealth effects and the impact of changes in the government expenditure are excluded (Figure 103).

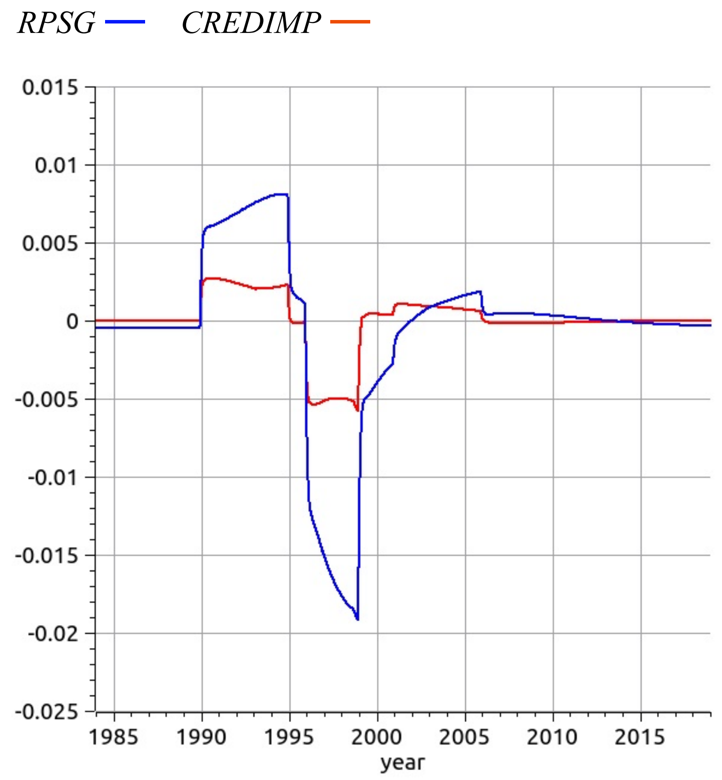

Figure 103: “HousingBubble”, credit impulse and changes in private spending.

4.15.3 Pseudo-Goodwin cycle

The Goodwin cycle is defined as an orbit in the employment – profit share phase (bivariate) space (Fiebiger, 2017). The original theoretical explanation provided by

Goodwin is the existence of a predator-prey mechanism, where the wage share is the predator and the rate of employment is the prey. This explanation has been rejected by a group of Post Keynesian economists, who point out the existence of an alternative mechanism of the business cycle. Fiebiger (2017) and Fiebiger and Lavoie (2017) point out the role played by Kalecki-Luxemburg's "external markets" and semi-autonomous household expenditures as the driving force of the business cycle.

The model can generate a pseudo-Goodwin cycle (Figure 104) because of the different lags affecting employment and profit share. The trajectory is traversed clockwise as the origin is at (0.95, 0.223) – these values correspond to the "Reference" scenario.

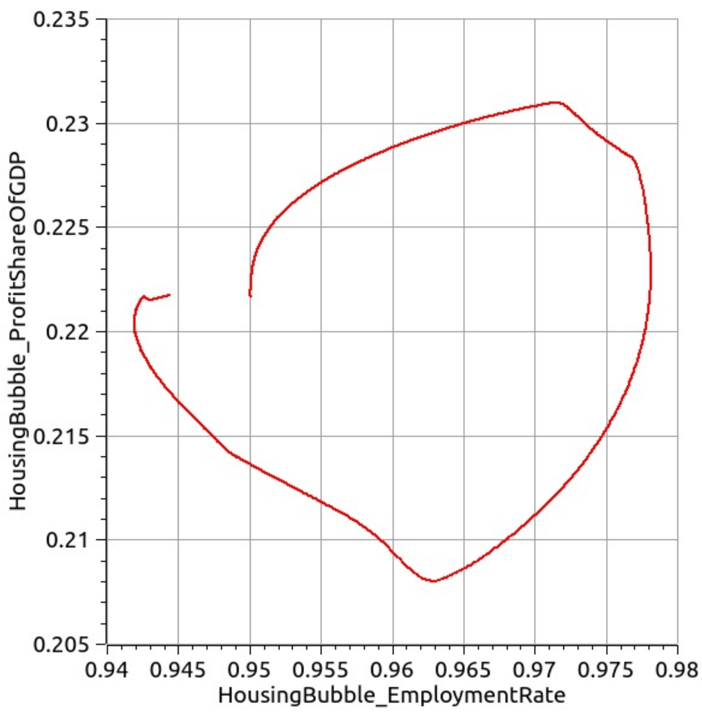

Figure 104: "HousingBubble" scenario, a pseudo-Goodwin (employment rate, profit share) cycle.

In the model, there is a significant lag in hiring workers (labour demand adjusts with a lag to labour demand target) while profits respond instantaneously to the volume of sales. This is illustrated in Figure 105 and Figure 106.

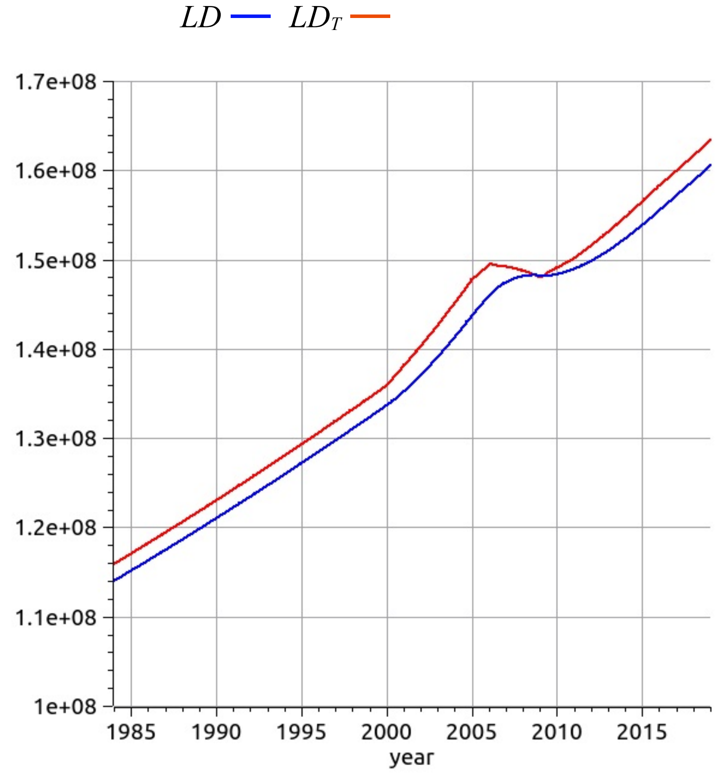

Figure 105: "HousingBubble", impact of labour demand target on labour demand.

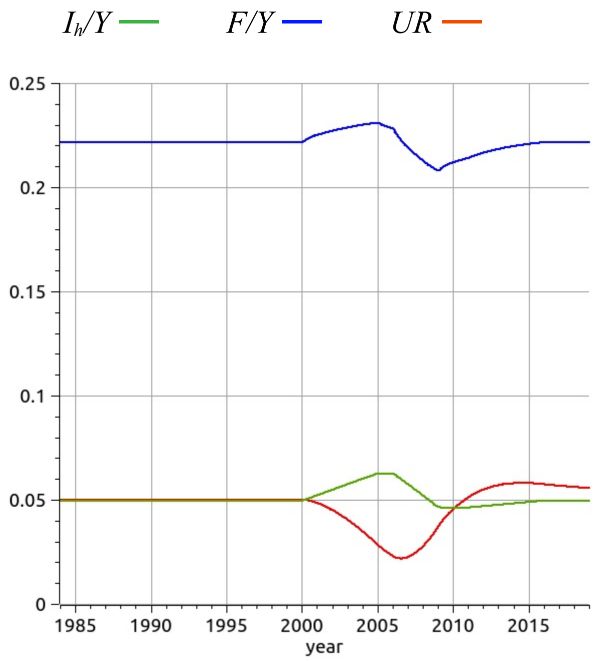

Figure 106: "HousingBubble", impact on housing investment on profit rate and rate of unemployment.

4.15.4 The Gibson paradox

According to Cogley, Sargent and Surico (2011), Keynes interpreted the correlation of nominal interest rates with the aggregate price level as contradictory to elements of neoclassical theory linking interest rates to expected inflation. The authors observed that the effect largely vanished between the early 1970s and 1995 due to the strong persistence of inflation in that period.

New Keynesian models have to be modified quite heavily to reproduce the phenomenon as they are built on the assumption that lower interest rates would lead to an increase in the rate of inflation (see 2.4).

The dynamic SFC model generates trajectories entirely consistent with the Gibson paradox. In fact the correlation is not a paradox at all in the context of cost-pricing. Higher interest rates increase firms' costs of servicing loans. These increases are passed through as the firms try defending the rate of profit (specifically, the value of planned gross profit) by increasing their markup, as described by equations (46) and (47). The process of adjusting prices upwards is seen as an increase in the rate of inflation.

The opposite process unfolds when there is a reduction of the rate of interest (Figure 107).

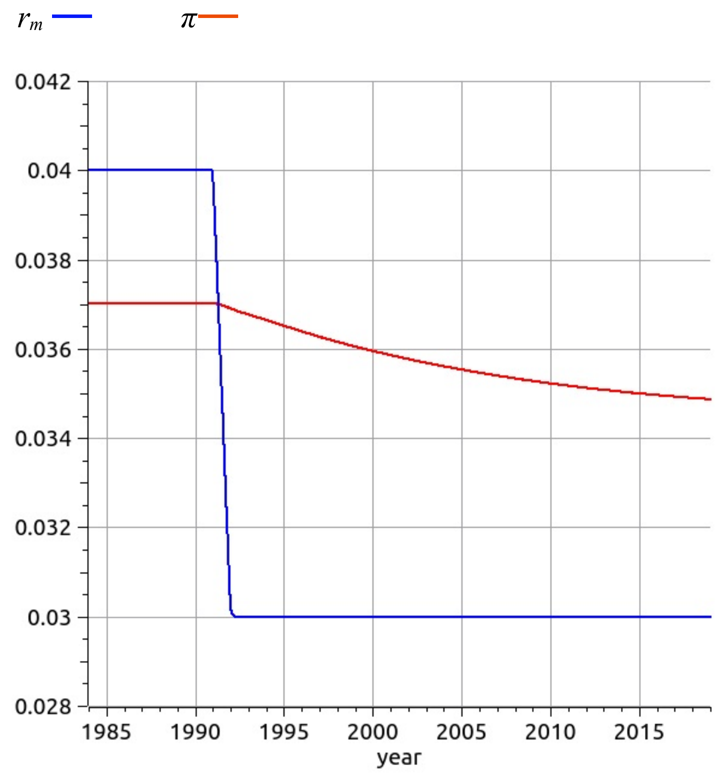

Figure 107: "MonetaryStimulus" scenario, the rate of interest and price inflation.

A reduction in r_m causes a similar fall in r_l. The firms costs pass-through mechanism is illustrated in Figure 108, when the cost of servicing corporate debt falls, the markup target is reduced. The actual markup follows the markup target with a lag. The reduction of the markup leads to the prices in the experiment scenario growing at a slower rate than in the reference scenario and manifests itself with a fall of the rate of inflation.

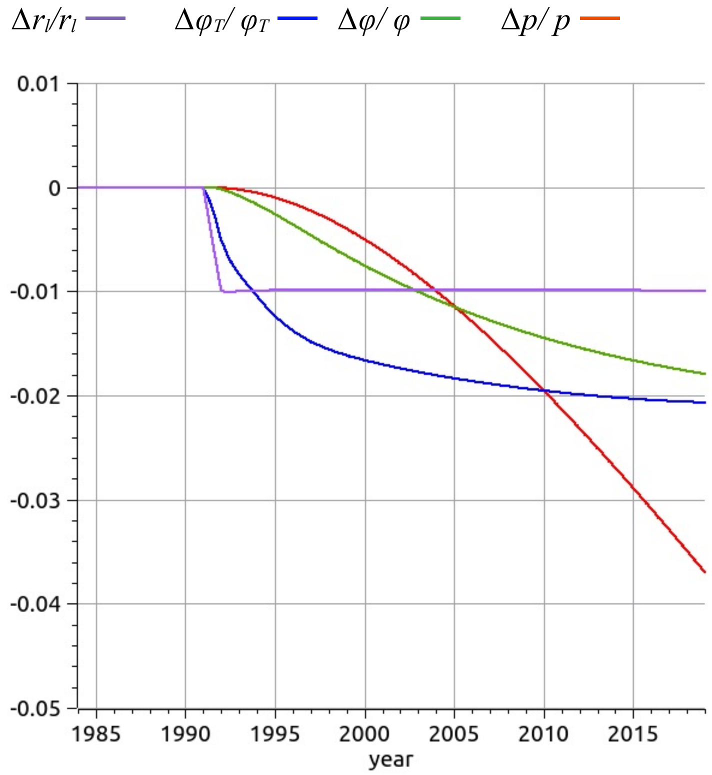

Figure 108: "MonetaryStimulus", the costs pass-through mechanism.

4.16 Discussion of the results of the simulations

4.16.1 Validation of the simulation of the GFC and Secular Stagnation

The model despite its simplicity can simulate a trajectory of GDP growth similar to the actual one (Figure 91). Despite the fact that the consumption function is based on simple behavioural assumptions (see Section 4.2), it is possible to build a realistic simulation of the changes in aggregate demand of the whole economy, driving the changes in the GDP. The goal of the modelling, which is to demonstrate that the GFC and Secular Stagnation can be causally explained by the changes in exogenous debt-financed household expenditure and the distributional changes between low-income and high-income households, has been achieved. Whether this causal explanation is correct or incorrect cannot be determined by building a dynamic SFC model but at least we have an alternative to DSGE models, based on more realistic assumptions and showing hysteresis and persistence effects without the need to introduce irrationality to the world of axiomatically rational representative agents.

As the Federal Reserve, during the GFC, managed to restore the functioning of the banking sector by lowering interest rates (Figure 8), providing enough liquidity and fixing prices of certain classes of mortgage based securities, a wave of bank runs affecting the real economy during Great Depression and resulting insolvencies destroying wealth of households was largely avoided. The GDP trajectory during the GFC can therefore be explained in terms of the collapse of residential construction investment (Figure 3 and Figure 83), negative wealth effects associated with falling house prices (Figure 6) and the collapse in business investment. These processes also led to a temporary reduction in the rate of productivity growth (Figure 86).

We do not need to get into the details of the processes developing within the financial and corporate sectors such as financialisation and globalisation. These processes undoubtedly played a major role in setting up the scene for the crisis but incorporating them into a simple dynamic model would lead to even greater complexity and could obscure the main message of Keynesian economics – a capitalist economy is demand-driven and governments can and should intervene by restoring full employment and sustainable economic growth if aggregate demand collapses for any reason.

For the sake of simplicity private real estate investment, land prices and to an extent business investment are treated as exogenous variables in the model. The simulation of the unemployment rate trajectory (Figure 85) is still qualitatively correct but less accurate. This is caused by the simplification of the job market where the participation rate is exogenous (but based on the actual data). Not explicitly introducing underemployment and with the corporate sector making only one good using two types of labour is all that can be achieved as the model of the job market is too oversimplified to allow for an accurate simulation of the participation rate. Several parameters such as the Gini index or the household saving rate show correct trends but their simulated values are offset from the true values measured in the American economy.

4.16.2 The effects of debt deflation in the real estate market on the whole economy

Keynes (1936, pp. 262, 264) argues that a reduction of money-wages, leading to a reduction of prices, will redistribute real income from workers and entrepreneurs to rentiers. Debt contracts are expressed in nominal terms and their real value increases as a result of deflation. The redistribution of income from workers towards the group having lower spending propensity and the increase of the real burden of debt will reduce both consumption and investment.

Debt deflation is defined by Fisher (1933) as an explosive feedback process in which deflation causes financial distress among agents who have accumulated excessive debt and financial distress amplifies deflation. Several channels through which this process affects the real economy can be considered. Fisher (1933) argues that what is affecting the GDP and resulting in price deflation is the contraction of money supply and falling money velocity.

According to Minsky (1975, p.125) the changes in the volume of investment were the main channel of propagating the disturbance from the asset market to the real economy:

> *...we are no longer in a boom; we are in a debt-deflation process. A feedback from the purely financial developments to the demand-for-investment output, and by way of the multiplier to the demand-for-consumption output, takes place. Unemployment and a depression result.*

Bernanke (2000, 2010) highlights the impact of defaults caused by the financial distress on the functioning of financial intermediaries, severely disrupting their operation, as mentioned in Section 2.5. Bernanke (2000) also explains that anticipated (incorporated into expectations of the agents) price deflation causes the real interest rate to rise (as the nominal rate cannot be negative), throttling firm investment.

Modelling the actual feedback loop driving the process of the growth of a debt-financed asset bubble and then subsequent debt deflation would require significantly extending the model and has not been attempted. These processes have been simulated by changing the values of control variables "η" and "REMOR" (as described in Sections 4.12.2 and 4.13).

Two major transmission channels from the financial and real estate markets to the real economy exist in the model:

- The fall in investment spending directly amplified by the supermultiplier.
- The fall in expected wealth of households directly reducing consumption expenditure due to negative wealth effects. This process can be seen as an increase in the personal saving rate, as discussed in Section 4.15.1.

A real estate bubble and subsequent debt deleveraging process have been simulated in "HousingBubble" and "HousingBubblePriceCrash" scenarios, described in Section 4.12.2. In order to isolate the impact of changes in the investment and wealth effects from other phenomena, the shocks have been calibrated in such a way that the response of the model remains linear. We can compare the size of the positive shock caused by the bubble, shown in Figure 55 (about 1.15% of the GDP) with the response of the economy, shown in Figure 61 (about 3.5% increase in the GDP). The amplification of the shock is significantly greater than the value of the short-term spending multiplier (supermultiplier), described in Section 4.9.1. The value of the fiscal multiplier is about 1.6. This discrepancy can be explained by the presence of significant wealth effects. These effects can be analysed by comparing the real estate wealth and GDP trajectories in "HousingBubble" and "HousingBubblePriceCrash" scenarios. They are depicted in Figure 57 and Figure 61. The only difference between these scenarios is the temporary reduction of real estate value in "HousingBubblePriceCrash" scenario. A fall in the wealth to GDP ratio by about 0.08 leads to a reduction of the GDP by about 0.02. This significant effect can only emerge in the model because the Modigliani aggregate consumption function depends on the expected wealth of high-income households. It would not appear in simple short-run Keynesian models of spending multiplier.

The historical scenarios simulate the actual large magnitude debt-deflation process in the real estate market, characterised by negative net mortgage lending (debt deleveraging, as shown in Figure 76) and negative growth of real estate prices (asset pride deflation, as shown Figure 77). Without the monetary and fiscal stimuli, the economy would have also experienced a period of significant price deflation, as shown in Figure 89.

The mechanism of the amplification of investment shocks based on the multiplier, originally identified by Minsky (1975), is linked with the observation that credit impulse affects the growth of private spending, as discussed in Section 4.15.2. The presence of the wealth effects and the impact of the changes in the government policies obscures the impact of the credit impulse on the growth of private spending in the historical scenarios.

4.16.3 The long-run spending multiplier

A Stock-Flow consistent model with a Modigliani-like consumption function will have its long-run trajectory of the stock-flow norm of wealth to GDP determined by the parameters of the consumption function and the long-run ratio of disposable income to GDP. An example of such a consumption function has been presented by Godley and Lavoie (2007, p.79), in the most basic model SIM.

$$C_d = \alpha_1 \cdot YD_e + \alpha_2 \cdot H_{h-1} \tag{171}$$

C_d is consumption demand, α_1 is propensity to spend out of disposable income, YD_e is expected disposable income, α_2 is propensity to spend out of wealth and H_{h-1} is stock of household wealth in previous period.

In this simple model variables are defined as nominal while more sophisticated models define real consumption, disposable income and wealth. The authors have demonstrated the following: when using reasonable values of the parameters, an increase in autonomous government expenditures by $20 will in the long run lead to an increase of the GDP by $100 (p.81, Table 3.6). The value of the short-term fiscal expenditure multiplier is 1.92 (p.70). Similar behaviour is shown by more complex models, including model GROWTH from Chapter 11.

The long-run response can also be described in terms of a government expenditure multiplier, accounting for wealth effects. The value of the long-run spending multiplier (about 5) does not seem to be realistic. Yet such is the logic of making consumption dependent on the stock of wealth and choosing reasonable at first glance values of spending propensities.

A more complex SFC model with a Modigliani-like consumption function calibrated to simulate the GFC and Secular Stagnation has the value of the long-run government expenditure multiplier between 2.5 and 3 (see Section 4.11.1). This seems still to be too high but might be more realistic.

Unfortunately there is no agreement about these values in econometric literature as neoclassical economists would like to see the value close to zero. The solution to this problem lies in building a more realistic aggregate consumption function, introducing multiple social classes (such as "the 1%") and making consumption out of the stock of expected wealth a non-linear function of wealth. Lumping together upper middle-class households and billionaires can only be considered a crude simplification.

Unlike model SIM (p.57) where the long run growth trajectory is determined by the growth of government expenditures, model GROWTH (p.378) is constrained in the long run by productivity growth. In this regard the model appears to be "supply-side constrained" like neoclassical growth models. However an actual growth trajectory is effectively determined by the demand side as long as there is less than full employment. In the long run government expenditure has to grow at the same rate as productivity and its level strongly affects the level of unemployment and capacity utilisation.

Brochier and Macedo e Silva (2017) describe the model as operating in the growth regime determined by government expenditures (like model SIM). Growth trajectories display hysteresis as it is the growth of the stock of wealth that directly determines the growth of household consumption in the long run.

If productivity growth is made partially endogenous (as it depends on improvements in technology and skills of workers which in turn depend on government research and development expenditure, corporate investment and the level of employment) then a long run growth trajectory at the technology frontier (the potential GDP) also displays hysteresis. A loss of growth opportunities is persistent (see Section 4.10). The behaviour of the model can be best described as "constrained endogenous growth".

It should be emphasised that in a closed economy the real aggregate demand (the sum of private and public sector real expenditures) cannot exceed the real potential GDP. The idea of a greater than zero elasticity of substitution in the real production function has been rejected. The production function used in an SFC model is effectively a Leontief-type function. The real GDP is constrained by the technology and the amount of available labour. A critique of the neoclassical aggregate production function (defined in the nominal domain) has been provided by Shaikh (1974).

The simulations demonstrate why the growth of the housing bubble had such a large and profound impact on the whole economy despite only being of the scale of a few percent of the GDP (Fiebiger & Lavoie, 2017). The initial trajectory can be explained by the Sraffian Supermultiplier magnifying the exogenous shock (a fall in investment of about 4-5% of the GDP). Assuming the value of the multiplier to be around 1.6, we can easily arrive at a realistic estimation of the dynamic response of the system in the short-run. Then the wealth effects kick in (the long-run multiplier seems to be higher than 1.6) and productivity losses caused by high unemployment shift the potential GDP growth trajectory downwards.

4.16.4 The evolution of public debt to GDP ratio in the long-run

Increasing the budget deficit to GDP ratio may under some circumstances lower the government debt to GDP ratio trajectory in the long-run. This has been demonstrated by Leão (2013).

The value of spending multiplier "m" (p. 452) defined as:

$$m=\frac{\partial Y}{\partial G} \tag{172}$$

has to be greater than the following ratio (p. 456)

$$m>\frac{1}{\frac{B}{Y}+\tau} \tag{173}$$

where B is the initial stock of government debt, Y is the GDP, G is government expenditure and τ (the effect of an increase in output on tax revenues, excluding government transfers) is defined as follows (p. 452):

$$\tau=\frac{\partial(T-T_r)}{\partial Y} \tag{174}$$

T are tax revenues and T_r are government transfers.

If the value of the spending multiplier is high enough, stimulating the economy reduces the government debt to GDP ratio (see Figure 98). Leão (2013) argues that this condition (173) is met by the economies of major developed countries.

These results invalidate neoclassical assumptions related to the No-Ponzi game condition for public debt. "I am not worried about the deficit. It is big enough to take care of itself" (Ronald Reagan).

Chapter 5 Conclusion

5.1 Addressing the research questions

The aim of this study has been to demonstrate how changes in parameters determining the behaviour of individual sectors of the economy and seemingly localised processes could explain the main macroeconomic phenomena affecting the whole American economy in the 21st century. The dynamic modelling framework allows for examination of the interactions between the components and for the determination of possible causal links, by reproducing them in the simulated environment.

All the research questions asked in Section 1.2 have been addressed.

The model clearly demonstrates the causal mechanism leading the economy towards stagnation if income inequality increases. If the government is not increasing its deficits and the balance of foreign trade does not change, the gap in aggregate demand widens and the economy enters "secular stagnation". This is a result of rising unemployment and a lower rate of productivity growth.

The transmission mechanism of both house and stock market bubbles is similar (it is an increase in investment and increase of the value of the assets, followed by a collapse of investment and fall of the value of the assets). A stock market bubble leads to an excessive increase in the stock of productive capital but its long-run consequences are more benign. A real estate bubble burdens households with an excessive stock of debt. A collapse of a housing bubble leads to changes in the distribution of disposable income between low-income households who have high marginal spending propensity and high-income households who have lower marginal spending propensity. The direct expenditure channel (a spending multiplier working in reverse) is the main transmission channel in the short-run, while negative wealth effects prolong and deepen the downturn in the medium-run. Finally, secondary effects such as productivity losses and a decrease of the participation rate reduce the potential GDP but in the long-run they allow for the employment rate to recover.

Post-Keynesian economists (Lavoie, 2014, pp, 345-346) argue that monetary policy is inefficient as a stabilisation tool especially if the ratio of public debt to GDP is high. According to Rochon and Setterfield (2017) the rate of interest is mainly a distributional variable. Based on the experience from the GFC it is assumed that a realistically calibrated model is not stimulated enough by a loose monetary policy during a recession. However, such policy is necessary to prevent an even deeper slump and a collapse of the banking sector. Expansionary monetary policy works in the model by redistributing income towards low-income households and by inducing moderate wealth effects due to the impact on the prices of shares. Some econometric studies suggest that distributional effects are weak and insignificant but at least in Australia (where the ratio of private debt to the GDP is high) the relevance of the household cash flow channel to monetary policy has been confirmed by La Cava, Hughson and Kaplan (2016).

On the other hand, fiscal policy is always effective in increasing aggregate demand and increasing employment. If the policy is only based on increasing government expenditure, social inequality remains high. Income redistribution policy can simultaneously stimulate the economy and reduce social inequality. Both variants of fiscal policy are equally efficient tools in reducing unemployment and restoring GDP growth. Even if the government stimulates the economy only by increasing its expenditure, there is no long-term risk of an explosive public debt trajectory and accelerating inflation in an economy resembling the United States of America, because of the wealth effects on high-income households. There might be a risk of social instability or instability of the global banking system if the rich are allowed to hoard too much wealth but these effects have not been examined.

The question facing economic decision makers is not whether to target the rate of inflation by fine-tuning monetary policy, using fiscal interventions only in emergencies, as in the New Keynesian world, but rather to choose the right kind of fiscal policy to achieve the desired level of production and allocation of real resources, while maintaining the environmental sustainability of the economy.

5.2 The significance of the contribution of the study

This study is intended to contribute to understanding of causal mechanisms affecting the economy at the macro level, by creating a relatively sophisticated theoretical model capable of reproducing several “stylised facts” about the economy of the United States of America.

The SFC modelling methodology has been enriched by building the model in the continuous-time framework, which should make future models easier to handle. The goal was to use a state-of-the-art modelling toolkit used in empirical and technical sciences (OpenModelica) to simulate relatively complex, nonlinear and dynamic macroeconomic processes. Bridging the gap between the methodology used in modern empirical sciences and in macroeconomics was one of the objectives of this study.

Macroeconomic models should not be treated as something entirely different to dynamic models used elsewhere. The inadequacy of the neoclassical and New Keynesian modelling approaches has been demonstrated in a very painful way, when austerity introduced after the GFC in Europe led to serious social problems.

The original Post Keynesian modelling framework created by Godley and Lavoie (2007) has been enriched by disaggregating the household sector to better capture the income distribution issues. High income households derive a significant portion of their income from labour, as managers and professionals. An increase in the fraction of the total labour income flowing to high-income households helps in explaining why the share of low-income households in the total disposable income has fallen more than the share of wages in the GDP.

Residential real estate assets have been added to the model to help simulate wealth effects, affecting the aggregate demand generated by the household sector. Debt-financed household investment, net exports and government expenditure are the main drivers of the economy in the short run.

While in the short run the changes in the level of economic activity are explained by the supermultiplier effect, as described by Fiebiger and Lavoie (2017), in the long run capital (wealth) accumulation and changes in the labour productivity determine the trajectory of economic growth. The role of fiscal policy in maintaining the adequate level of aggregate demand has been highlighted.

The goal of this study was to provide a simple but comprehensive theoretical model of the whole economy which is more realistic than DSGE models, even if it has not been empirically calibrated.

5.3 The limitations of the methodology and the model

It is obvious that the SFC methodology has its limitations. It does not allow for simulating emergent phenomena arising at the microeconomic level; macro behaviour has to be assumed. The model has to be controlled by the use of exogenous parameters in order to simulate changes in human behaviour such as changes in the portfolio allocation function or increased demand for housing assets. We can endogenise these processes but there is always risk of shoehorning more complex processes into a familiar predator-prey model, which would generate cyclical oscillations. This approach is very narrow in a system which has several degrees of freedom in the short-run and demonstrating that the model can generate periodic oscillations does not prove that the sources of instability have been correctly identified.

It is not valid to exclude the impact of other variables such as consumer confidence or the willingness of the banking sector to extend credit because these variables are absent in the model and the model has generated a realistically looking trajectory. We can only argue that the crude explanation of the business cycle by changes in semi-autonomous household expenditures has been supported by econometric studies, as demonstrated by Fiebiger (2017).

Another set of limitations is related to the actual implementation of the model. The finance sector is presented in a very simple way and the process of financialisation has been ignored. The productive sector only makes one good and the labour market is highly oversimplified. The interactions of the domestic economy with the foreign sector have been reduced to net trade balance. All the processes linked with globalisation have been ignored. These limitations have been acknowledged in the thesis but it can be argued that the main goal, the simulation of the GDP trajectory, has not been compromised.

Finally, the calibration procedure used in this study is quite crude. The model could be used as a scaffolding to build empirical SFC models.

5.4 Future research recommendations

The model introduced in this study can be developed further by adding multiple sectors producing many commodities and using several techniques, which could allow for modelling the ecological footprint of the economy. The household sector can be divided into more income groups and the consumption function can be made more realistic, by introducing non-linearity in the aggregate behaviour of consumers and allowing the parameters to change in time. The labour market model can be developed further by introducing discouraged workers as a separate group and specifying transition probabilities. The foreign sector may use separate currencies and trade multiple commodities. It is possible to make all these changes using the modelling framework and toolkit (OpenModelica) already introduced in this study. Finally, introducing a graphical representation of the model can make it more accessible to other researchers.

Appendix

Source code of the model

Model source code has been licensed under the GNU General Public License version 3. The code is available in the following location:
https://github.com/Adam-Kaczynski/sfc-pk-gfc-model

The scenarios can be simulated using OpenModelica, available from:
https://openmodelica.org/

The definition of the baseline model and all scenario simulations are stored inside "EconomicModels.mo". This file can be opened and simulated using OMEdit but it is not possible to store the definitions of the graphs created during simulation in order to recreate them if the model definition or simulation parameters are changed. OMEdit is integrated with a text editor, allowing to modify the code.

Graphs are defined in a notebook-like environment, OMNotebook. This tool is less convenient for modifying the code. These scenarios which can be simulated between 1984 and 2019 are included in "SfcModelOfGfc.onb" while long-run scenarios simulated between 1984 and 2084 have been separated onto "LongTermGrowth.onb". Some short-run scenarios are not stable in the long run.

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